GRAUTEN

Investigation of
Stanley Induction
Watt-Hour Meter

Electrical Engineering

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INVESTIGATION

OF

STANLEY INDUCTION WATT-HOUR METER

BY

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THESIS

FOR

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IN

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

SYLVESTER HENRY GRAUTEN

ENTITLED INVESTIGATION OF THE NEW STANLEY WATTMETER

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE

OF BACHELOR OF SCIENCE IN ELECTRICAL ENGINEERING

Morgan Brooks.
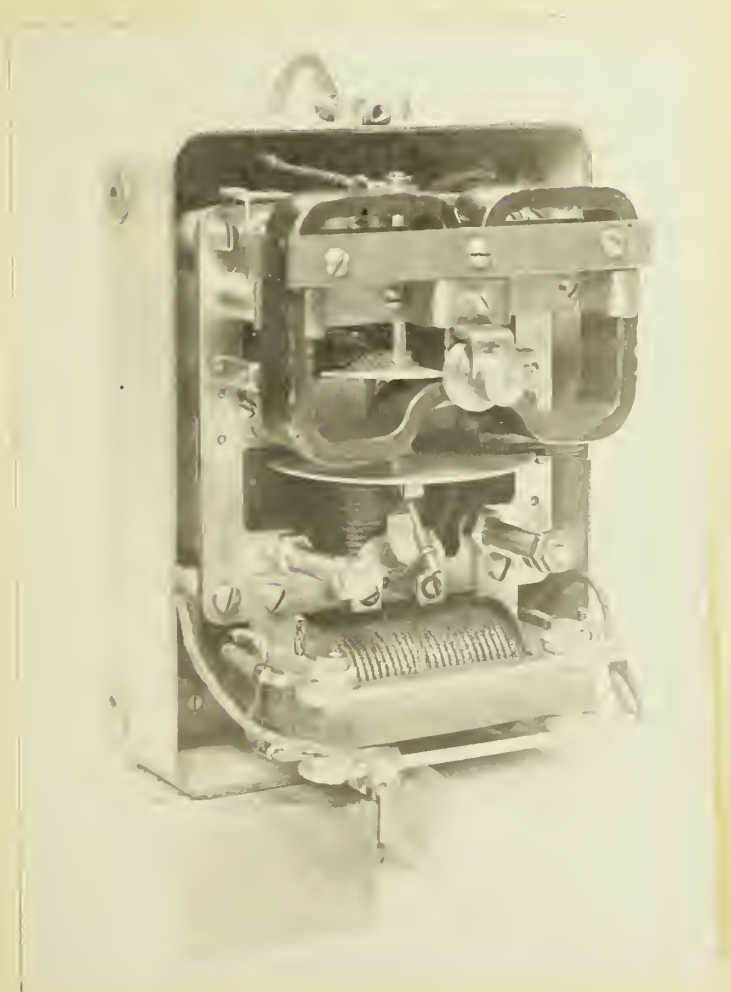
HEAD OF DEPARTMENT OF ELECTRICAL ENGINEERING

Contents

Introductory.

The extensive use of alternating currents in the electrical field may be ascribed to an inherent property of such a current. It is the phenomena of an ever-changing field which encircles a conductor carrying an alternating current, that allows of the flexible transformation, both of current and energy. This property of "action at a distance" allows of a great simplicity of transformation which may be to a stationary secondary or to a revolving rotor. The application of this fundamental principle may be summed up broadly under the general alternating current transformer.

The inherent advantages of this type of apparatus apply equally to the "induction meter where the property of high torque and consequent accuracy on light load, coupled with their simplicity of construction and low cost of manufacture, make them an ideal piece of apparatus.

The invention of a new device which does not infringe upon these patents of Tesla and Shallenberger must of necessity embody features which are both original and radical. This, however, is only a part of what might be claimed for the new induction wattmeter, invented by William Stanley. Here we have a novel idea in a practical form which is strictly in accord with what experience has shown to be the best meter practice.

The new principles involved in the motor device, may be summarized as follows:- ✳

✳Suggestions due to Mr G.Faccioli.

(1) No shunt flux separate from series flux.

(2) Each motor has but _one_ energizing flux.

(3) Torque _not_ due to reaction of eddies excited by shunt
flux upon series flux and vice versa but due to reaction
of eddies upon a new flux that they themselves produce;
therefore torque is due to reaction of eddies and flux
which are _always in phase_.

It is seen that the principles embodied are novel and it
might be expected that the application of these principles
would involve further innovations.

In fact the application of the fundamental principle of
the "self reaction" of a current on its own flux, presented
several distinct problems. Of these, we have the solutions in
the wing motor, the differential system and the combining trans-
former. The whole representing a unique solution of a diffi-
cult problem; one that is concise, clear cut and logical from
a mathematical point of view; and an efficacious one as regards
the patent situation.

In the present work it has been the purpose to make the
work of as general a character as possible and not to make a
commercial test of the experimental model the basis of the in-
vestigation. The work was found to be intensely interesting
and was with regret curtailed on account of the limited time.
Of the experimental data but a small portion is presented.

Thanks are due to the inventor, Mr William Stanley, for
the loan of the experimental model and to Mr G.Faccioli for his

patience in explaining the principles of the device and to Professor H.E.Clifford, by whose courtesy the work was performed at the Massachusetts Institute of Technology.

Boston, May 6,1907.   S.H.G.
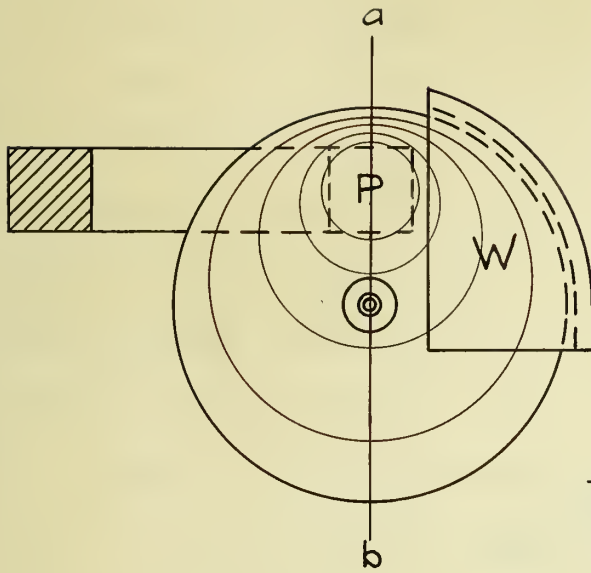
Top View :— Upper pole
cut away, showing
eddy current paths
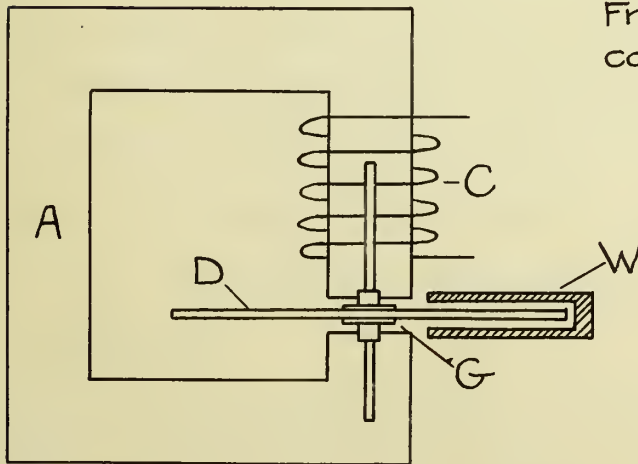and position of wing.

Front View :— Showing
core, disc and wing.

MOTOR ELEMENT. (Figs. 1 & 2).

2. General Description.--

Motor Element. The torque producing or motor element of this new
type of meter consists, essentially, of two electrical and two
magnetic circuits. The two will be called the primary and se-
condary electrical (or magnetic) circuits, respectively. The
primary or energizing circuit consists of a coil,C, (Fig.1) placed
on the pole of the laminated iron core,A. This core which con-
stitutes the primary magnetic circuit, has a gap,G, through which
passes the edge of the disc,D. This disk comprises the secondary
electrical circuit and is enveloped in part by the stationary,
iron wing,W, placed on one side of the pole,P. This wing com-
prises the secondary magnetic circuit.

Action. An alternating current flowing in the primary coil produces
          a flux in the gap,G. This alternating flux induces in the
disc, eddy currents (the secondary current) whose paths are cir-
cles with centres on the diameter, a b, through the centre of the
pole,P. Obviously the torque effect of these currents on the main
flux is nil, as owing to the symmetry of the arrangement, the
action on the right will be equal to that on the left and in the
opposite direction.

          The wing on the pole,however, closely envelopes the secondary
current and produces on the one side of the pole a low reluctance
path for the secondary flux. This flux is acted on by the se-
condary current and there results a torque in the direction of the
wing.

It is in this basic property that the device differs essentially from all other devices of its kind. The active flux being produced by the active current is necessarily in phase with it and we have an action independent of the phase relations of the primary and secondary (or eddy) currents, a principle at once novel and of great importance.

Law of Meter. The torque produced by the current, I_2, in the field, ϕ_2, is evidently proportional to the product of these two quantities

$$T \propto I_2 \phi_2$$

where, T, is the torque and, I_2 and ϕ_2 are the secondary current and flux respectively. The flux, ϕ_2 is produced by the current, I_2 and it follows

$$\phi_2 \propto I_2$$
$$\therefore \quad T \propto I_2^2$$

It will now be shown that the current, I_2 is proportional to the primary current, I_1. The current, I_1, in the coil, C, produces the flux, ϕ_1, and it is proportional to it. It then remains to prove that the secondary current is proportional to the primary flux in order to establish the desired relation. The metallic disc may be considered as a coil of one turn, if E_2 be the effective value of the electromotive force of this circuit

$$E_2 = 4.44 \phi n \ , \ \text{\&} \ E_2 \propto \phi_1$$

where, n, is the frequency and, ϕ_1 is the primary flux. A form factor of 1.11 being assumed.

Mains.

Load.
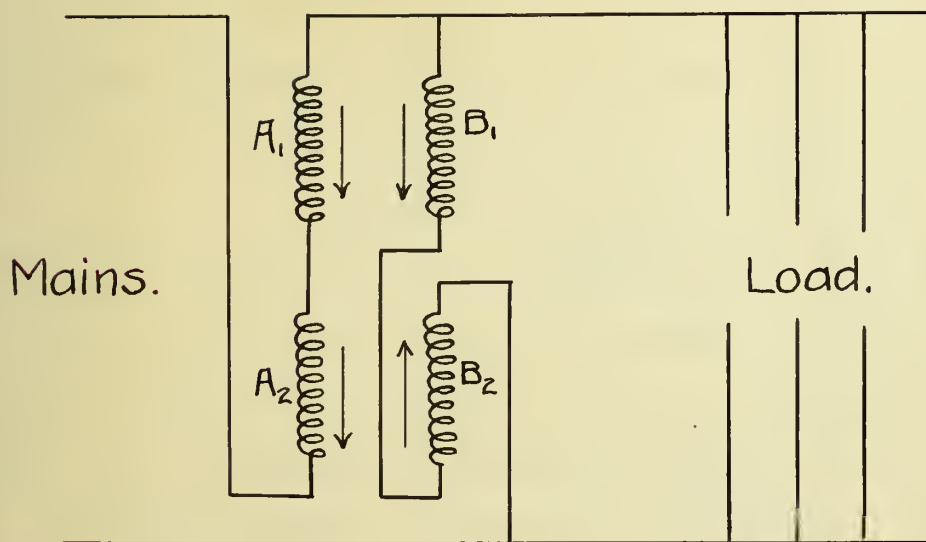
DIFFERENTIAL SYSTEM. (Fig. 3).

$$I_2 = \frac{E_2}{R_2}$$

$$\therefore I_2 \propto E_2 \propto \phi_1 \propto I_1$$

$$T \propto E_2^2 \propto \phi_1^2 \propto I_1^2$$

also $T \propto E^2 \propto n^2 \propto I_1^2$

Differential System. It has just been shown that the torque of
the motor element it proportional to the
square of the current in coil,C. It is de-
sired however that the rate of the meter be directly proportional
to the currents supplied; it is therefore necessary to "auto-
matically extract the square root" of this torque. This is ac-
complished by means of two elements working differentially.

The two elements are identical except that the wing on the
lower one is placed to the right while that on the upper is to
the left. Now the two discs are mounted on a common shaft and
as the two elements work in opposite directions or in "differen-
tial", we have a net torque equal to the difference of the two

For the measurement of power each element is equipped with
two windings. One of these carries a current proportional to the
main current and the other a current proportional to the e.m.f.
of the circuit in which it is desired to measure the power input.
These windings are arranged as in Fig.3 and it is seen that, while
the current, A, in the coils A, and A_2 flows through each in the
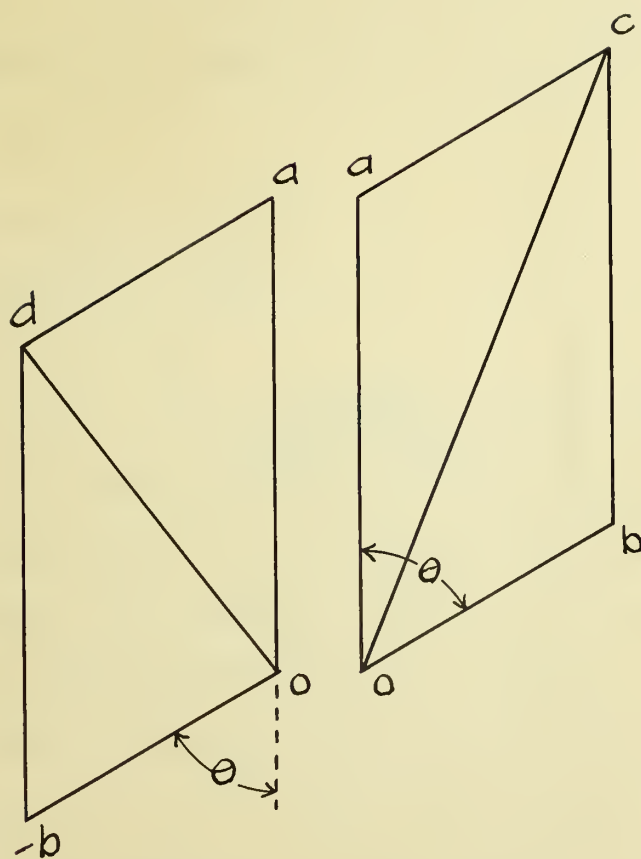
VECTOR   DIAGRAM. (Fig. 4)

same direction, the current,B, flows through B_1 and B_2 in opposite directions, All coils being wound in the same direction.

The current **A** will produce a flux,a, in the gap and the current, B, a second flux,b. These fluxes combine and we then have in the upper gap, a resultant, (a+b), and in the lower, a resultant (a-b). It has been shown that the torque in the motor element is proportional to the square of the primary flux. It follows

$$T_1 = (a+b)^2 = a^2 + 2ab + b^2$$
$$T_2 = (a-b)^2 = a^2 - 2ab + b^2$$
$$T = T_1 - T_2 = 4ab.$$

As the fluxes,a and b, are proportional to the currents, A and B, respectively, it is seen that we have a device in which the torque is directly proportional to the product of the two currents and if A and B represent the current and e.m.f. in a circuit we have a measure of the power expended in that circuit.

In a case where the power factor is not unity, the input will be E I cos Θ. It may be shown that the rate of the meter is proportional to this quantity on such a load. The two fluxes a and b will no longer be in phase but at the same angle as their corresponding currents, **A** and B. Their resultant will now be the vector sum.(Fig.4) in the one case and the vector difference in the other. It follows

$$T_1 = c^2 = (a+b\cos\Theta)^2 + (b\sin\Theta)^2$$
$$T_2 = d^2 = (a-b\cos\Theta)^2 + (b\sin\Theta)^2$$
$$T = T_1 - T_2 = 4\, ab\cos\Theta$$

It is in this differential system that the important principle

Mains.

Load.
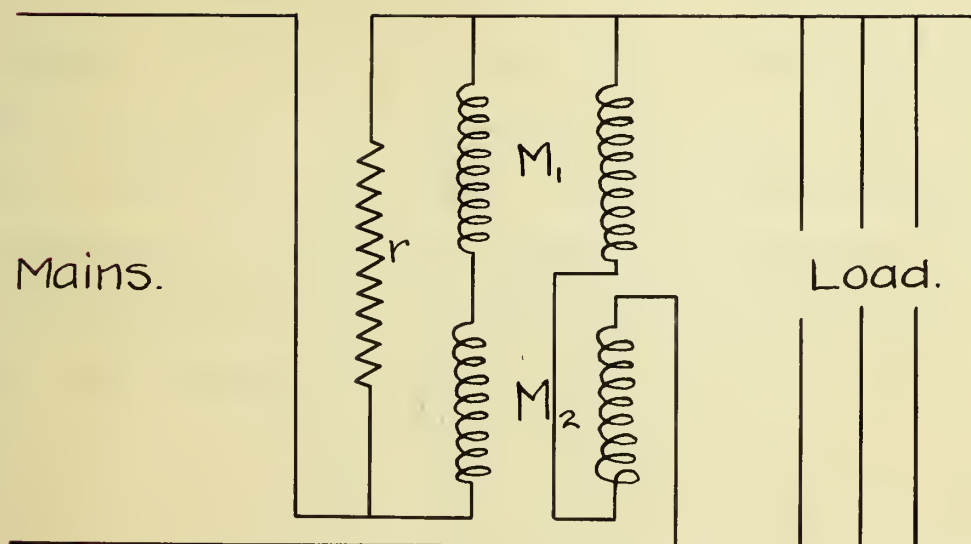
COMPENSATED ARRANGEMENT. (Fig.5)

already stated is utilized. The current and e.m.f. combine to produce a single flux which produces the secondary (active) current which in turn produces the secondary (active)flux. The action of the element depending on the simple reaction of these two and the action of the system depending on the mechanical combination of two such elements.

Compensation. It has been shown that the secondary current depends not only upon the flux but also upon the frequency of the supply. It is therefore necessary to compensate for variations of frequency. This is accomplished as shown in Fig.5. The coils B_1 and B_2 are shunted by a noninductive resistance,R, and the resistance of the coils is made relatively small. The current in the windings of the meter will then vary inversely as the frequency and directly as the line current or

$$E_2 \propto \phi n$$
$$\phi \propto I_1 \propto \frac{1}{n}$$
$$\therefore E_2 \propto \frac{1}{n} \times n \propto I$$

The torque produced will then be practically independent of frequency.

The constants of the shunt and series coils can be arranged so that the current in each lags behind its corresponding vector by the same amount.
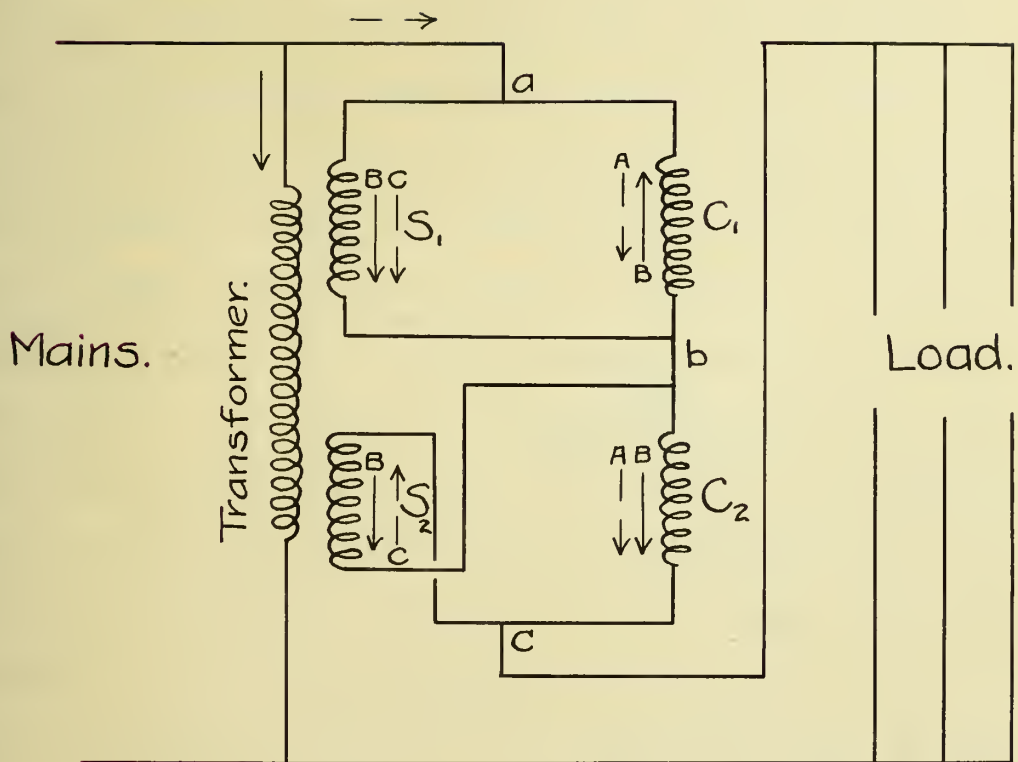
MODIFIED ARRANGEMENT. (Fig. 6).

Modified Form. In the foregoing arrangement, two coils were

necessary on each pole, to provide the current and

e.m.f. factors, respectively. It is possible, however, to combine

these two currents in a single coil to produce the same result.

This is accomplished by the arrangement shown in Fig.6. M_1 and M_2

are the motor elements, identical with those used previously but,

as above, provided with but one coil. The transformer is arranged

with two secondaries which are connected as shown. The primary is

connected directly across the mains and induces currents in the

two secondaries which are closed through the motor elements, C_1 and C_2

respectively. The line current, I, enters at, a, and with the se-

condary, S_1, as a shunt circuit, passes through C_1 when it reaches

the point C. It again splits at, d, and similarly unites at, b. It

will be noted that as before the current, I, flows through both

coils in the same direction while the terminals of the lower se-

condary are reversd, so that the e.m.f. <u>acts</u> in opposite directions

in the two coils, <u>a condition</u> identical with the differential sys-

tem of the earlier form. If we denote the line current through

C_1 & C_2 by I_1 and that through the secondary S_1 & S_2 as I_2 , it is

evident that I_1 will be proportional to I. We have the difference

here from the early form that the <u>currents</u> A and B now unite and

produce a flux proportional to, (A+B) or (A-B), while before the

fluxes only were united.

It is evident that the component of, I, which flows through the

elements C_1 & C_2 and which will be denoted (as before) by, A, will

be proportional to the main current, I. Also, the current produced

in the elements due to the e.m.f., E, and denoted by, B, will be
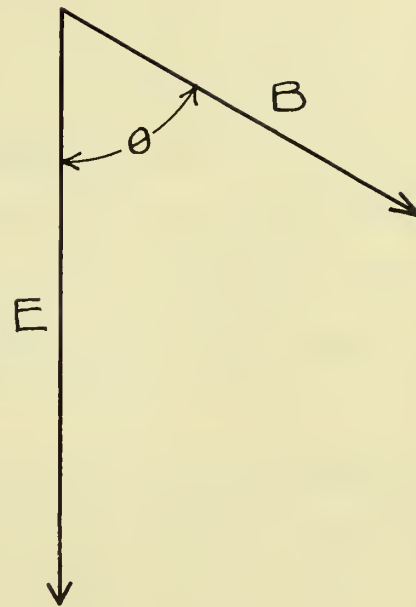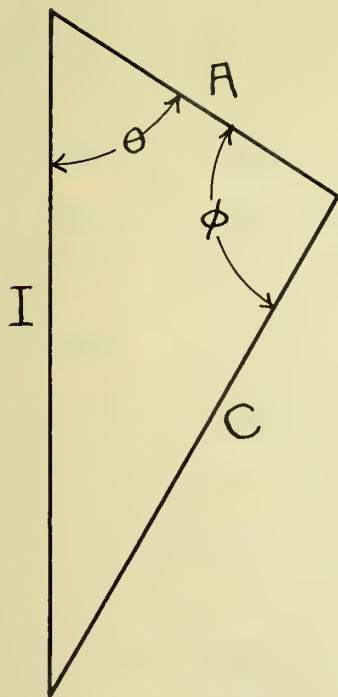
VECTOR DIAGRAM. (Figs. 7 and 8).

proportional to,E. There will then be a resultant current,(A-B) in the coil C, and a resultant,(A+B), in the coil,C_2. These currents will produce the primary fluxes,(a+b) and (a-b),respectively.

It is necessary,however, to consider the phase relations of the components, A and B. The line current,I, is resolved into the components A and C (Fig.7), the angle,Θ_1,between the current,I, and the component,A, will depend on the constants of the two circuits. Considering the e.m.f. factor, the current,B, will lag behind the impressed e.m.f.,E, by an angle Θ_2. It is obvious that these two angles, Θ_1 and Θ_2 must be equal, as if the current,A, does not lag behind, the line current, I,by the same amount that the current,B, lags behind the e.m.f.,E, the true vector sum (A+B) and (A-B) will not be attained.

It is easily proven that the desired relation exists in the arrangement as employed here. Consider the simple series circuit, S, b c, a(or S_2bC_2c), in which a current flows due to the e.m.f. of the transformer secondary of the resistance of the coil,C, be considered as negligible compared with its reactance,X, and if in the coil,S, the reactance be small compared with resistance,V, we have a simple series circuit of resistance,V, and reactance,X, and

$$\tan \Theta_2 = \frac{X}{R}$$

In the parallel circuits, a S b and a c b, (Fig.7) we have

$$A = \frac{E}{X}$$

$$C = \frac{E}{R}$$

the angle ϕ is evidently a right angle

$$\frac{C}{A} = \frac{X}{R} = \tan \Theta_1$$

But $\tan \Theta_2 = \frac{X}{R}$, therefore $\Theta_1 = \Theta_2$.

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3. Analytical Discussion.

A general outline of the action and principle of the meter has already been given. The more detailed study of these actions is interesting as it bears directly on the design of the meter, a few of these considerations will be briefly mentioned.

Primary Flux. The primary flux is directly produced by the primary current with which it is in phase. As will be shown later the reaction of the secondary on the primary is negligible. We then have a simple law between primary current and flux. This follows from the large air gap included in the magnetic circuit, of which it contributes about ninety per cent of the total reluctance and also from the low flux densities employed in the iron. If, A, is the area of the pole and, l, the length of gap and, ϕ_1 the primary flux then,

$$\phi_1 = \frac{4 \pi n I A}{11.l}$$

It also follows from this equation that the flux curve will be similar in form to the current wave.

Secondary E.m.f.   The alternating flux, ϕ , produces an e.m.f. in the disc, which is given by the formula

$$E_2 = 4 f N \phi \times 10^{-8}$$
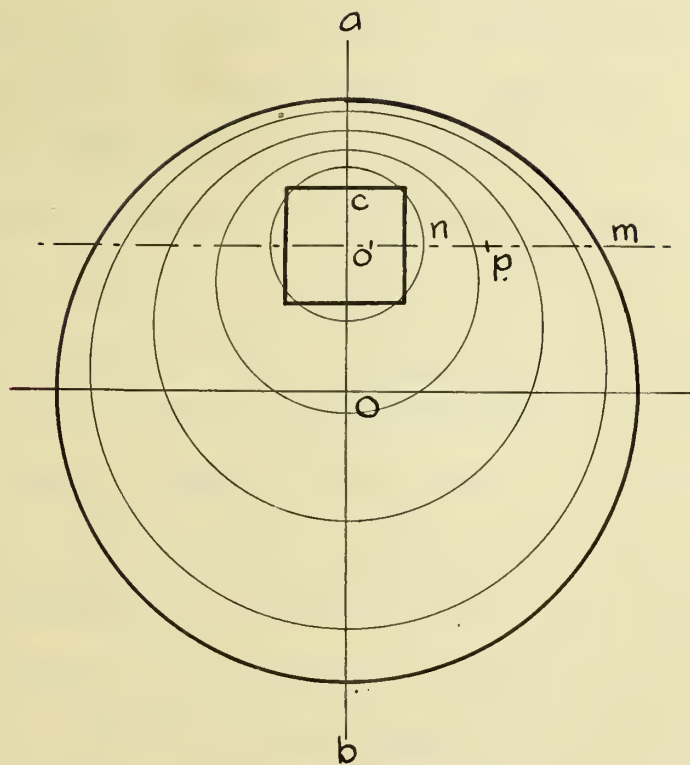
EDDY CURRENTS IN DISC. (Fig. 9).

where,f, is the form factor and ,N, the cycles per second.

Secondary   The secondary circuit consisting of the conducting
Current.   sheet or disc is the seat of the "eddy currents" upon
           which the action of the device depends. These "currents"
are more properly a single stream varying in density according
to the distance from the pole. The stream lines of Fig.9 are
drawn through points of equal current densities.

     If a uniform field be assumed throughout the pole face, it
is evident that no current can flow under the pole face itself.
For at any point within the field, Fig.10, there will always be
two equal and opposing e.m.f.s which neutralize each other. From
this consideration it follows that all the current must flow be-
tween the pole and the edge of the disc; the narrow strip, a c,
Fig.9. As the current density is necessarily much greater in the
narrow strip through,a c, it follows that the drop in potential
through any element of the disc at the point be correspondingly
greater than one situated diametrically opposite. It is seen the
position of the pole has a direct effect on the secondary resis-
tance. The distribution throughout the disc will be in accordance
with Ohm's Law and the stream lines will be a series of circles
having their centres on the diameter, a b. As the resistance of
any such path will be directly proportional to its length and
consequently its radius, it follows that the current density
will be inversely proportional to the radius of the path. The in-
ductance of the circuit may cause a slight variation in the as-
sumed distribution but it will be neglected here. It follows

ELECTROMOTIVE FORCES UNDER POLE FACE.
(Fig. 10.)

that the resistance of all paths will vary inversely as the thickness of the disc. The torque will obviously be effected by the square of this dimension.

If the current density at any point be denoted by, d, and, V, be the distance of the point from its centre and, w, be the width of the disc at that point, as, m n, Fig.9. it follows that the density at any point will vary inversely as the radius and also as the total width, w, then

$$d = \frac{K}{wV}$$

K, being a constant depending on the e.m.f. and thickness and conductivity of the disc. Fig.11 shows the distribution over a section as, m n, Fig.9.

It will be convenient in the following to locate the point in mid-current, ρ ,i.e. the point at which the current integral to the left, n p, is equal to that on the right, p m. In Fig.11 it is evident that the area of the curve between any two radii represents the summation of the current between these limits as

$$dA = ydx \quad \text{but } y = d = \frac{dI}{dr}, \quad dx = dr = \frac{dI}{dr} \times dr = dI$$

$$\therefore \quad I = \int ydx = \int ddr = \int \frac{K}{v}dv = K \log \frac{r}{c}$$

The value of c may be determined by evaluating for $r = 0$ $n = r_0$, when, $I = 0$, $\quad \therefore \quad c = r_0$

Inserting the limits between which we wish the summation

$$I = K\left(\log \frac{r}{c}\right)_{r_1}^{r_2} = K \log \frac{r_2}{r_1}$$
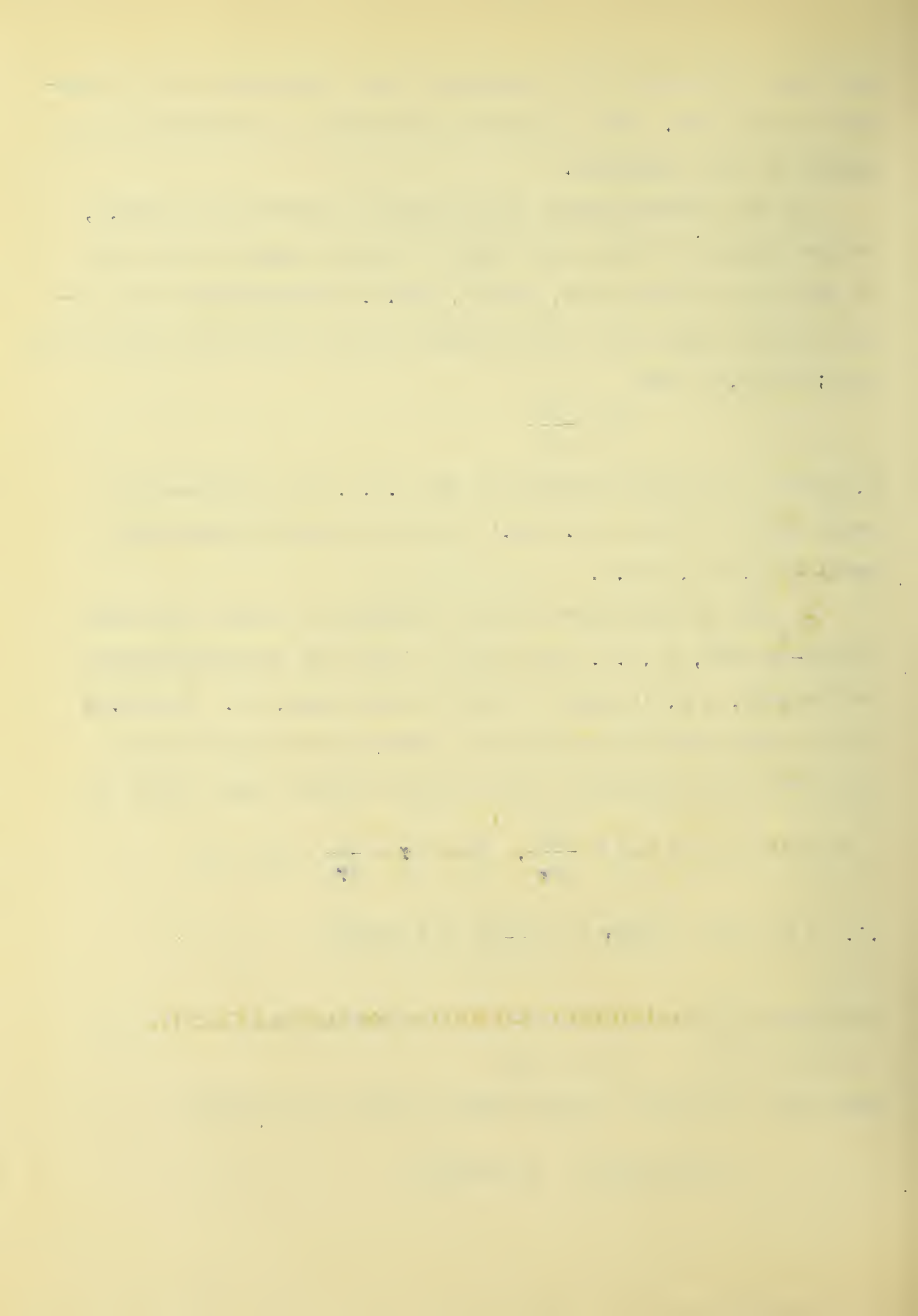
CURRENT DENSITY IN DISC. (Fig.11).

Then if, r_1, be the inner radius, r_3, the outer radius and, r_2, the desired midpoint.

$$K \log \frac{r_2}{r_1} = K \log \frac{r_3}{r_2}$$

$$\frac{r_2}{r_1} = \frac{r_3}{r_2}$$
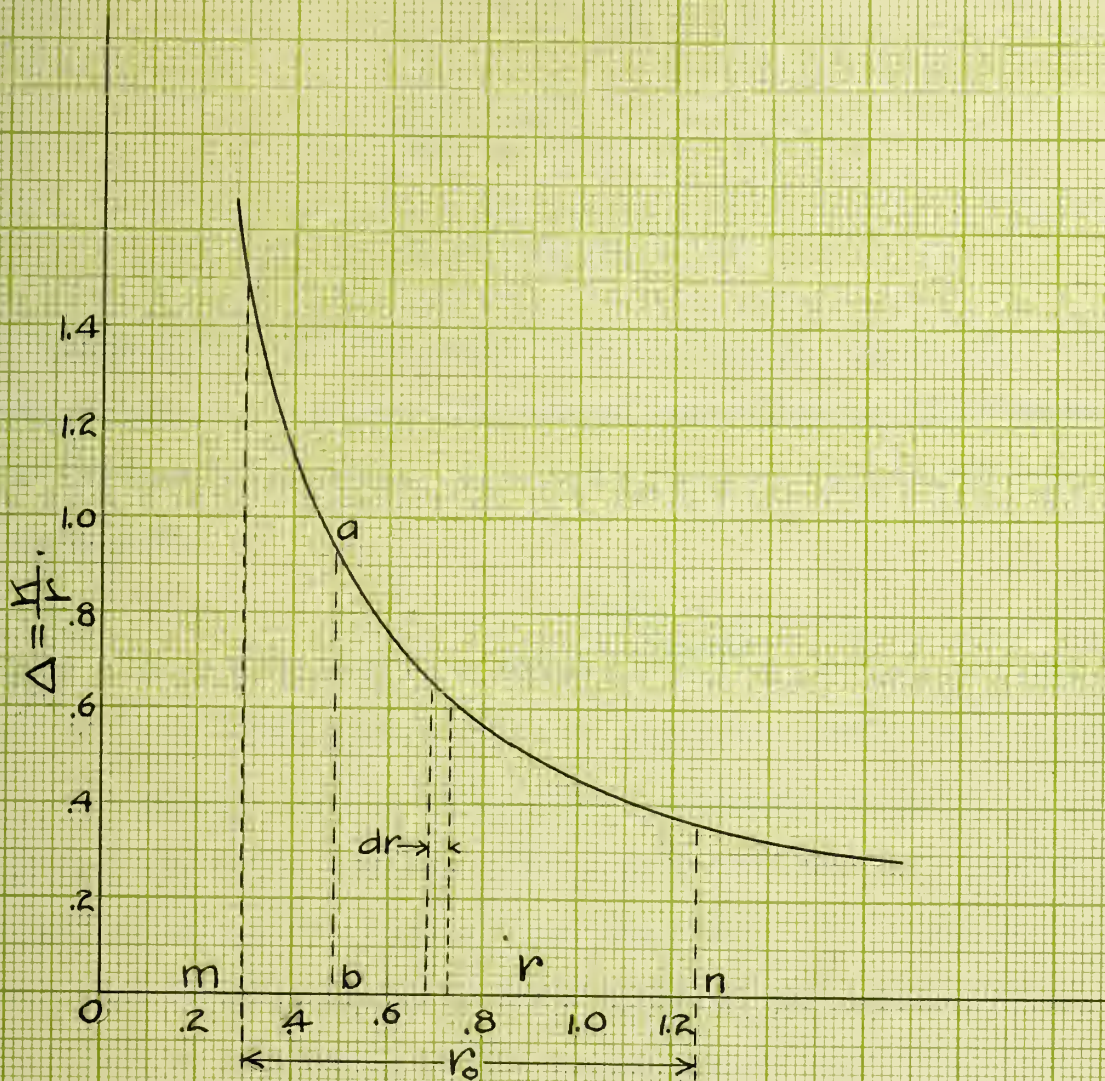
$$r_2 = \sqrt{r_1 \, r_3}$$

This value might also be taken from the curve, Fig.12, the ordinate at any point representing the current enclosed by the stream line drawn through that point. The distance oc on this curve represents the radius r_0

Secondary   The secondary flux which is set up by the current in the
Flux.   disc will depend for its magnitude and path upon the
              magneto motive force and reluctance of the position.
There are two paths however which represent the most likely cases.
We might have a flux going around the path, g c b, Fig.13. This
is unlikely however owing to the length of air gap included, al-
though such a flux would have a large m.m.f. as it would link with
the maximum current. Again the flux might take the path, d a, this
would include only a single air gap but would have the m.m.f. of
the heavy current outisde of the section, d a. If the point of
mid current be at, p, there will be no effective m.m.f. outside of
this point. Any point inside however will possess an m.m.f. and
moreover the resultant flux will cut a large current and obviously
will be useful in producing torque.
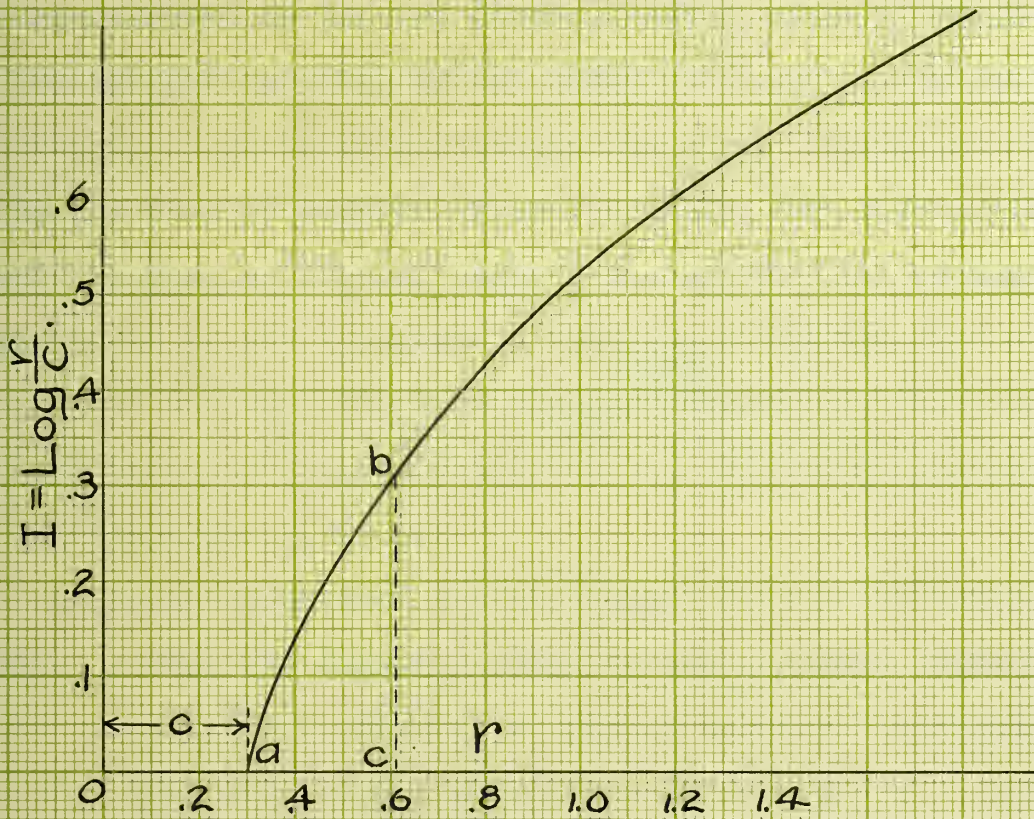
Current Integral. (Fig. 12).

A simple experiment will show that the second hypothesis is the correct one. It is observed that the direction of rotation is always toward the wing. Now a consideration of Fleming's rule will enable us to determine the direction of the effective flux and subsequently, its path. Assume the current,I, Fig.14, to be flowing in the direction shown,at some instant; the two cases discussed are represented by the two fluxes,X and Y, each of which is caused by the current enclosed by it. Each of these is seen to cut the current,I, in opposite directions and consequently would tend to force the disc in reverse directions. Now, the disc revolves towards the wing as already stated, A consideration of the above mentioned"rule of the thumb"shows that the effective flux at this instant must cut the disc in a downward direction, therefore the flux Y which fulfills this condition is the effective flux and the path is through,d,and,a,Fig.14.

A second experiment was previously made to determine the same point. A wing was made according to the standard form but was built so that the section at,a, Fig.13 could be interchanged and the wing used with either a brass or an iron piece at this point. It is seen that if the path of the secondary flux were according to the first assumption, i.e. through g.b. the effective flux would be diminished if the section at,a, were of iron and increased if of brass. Now, if the path, d a, is the effective one, the torque will be effected in the reverse manner as the brass piece would introduce an additional reluctance in the path, and the torque will be diminished.

Current-speed curves were taken in the manner explained elsewhere, for the wing equipped first with an iron spacer and then
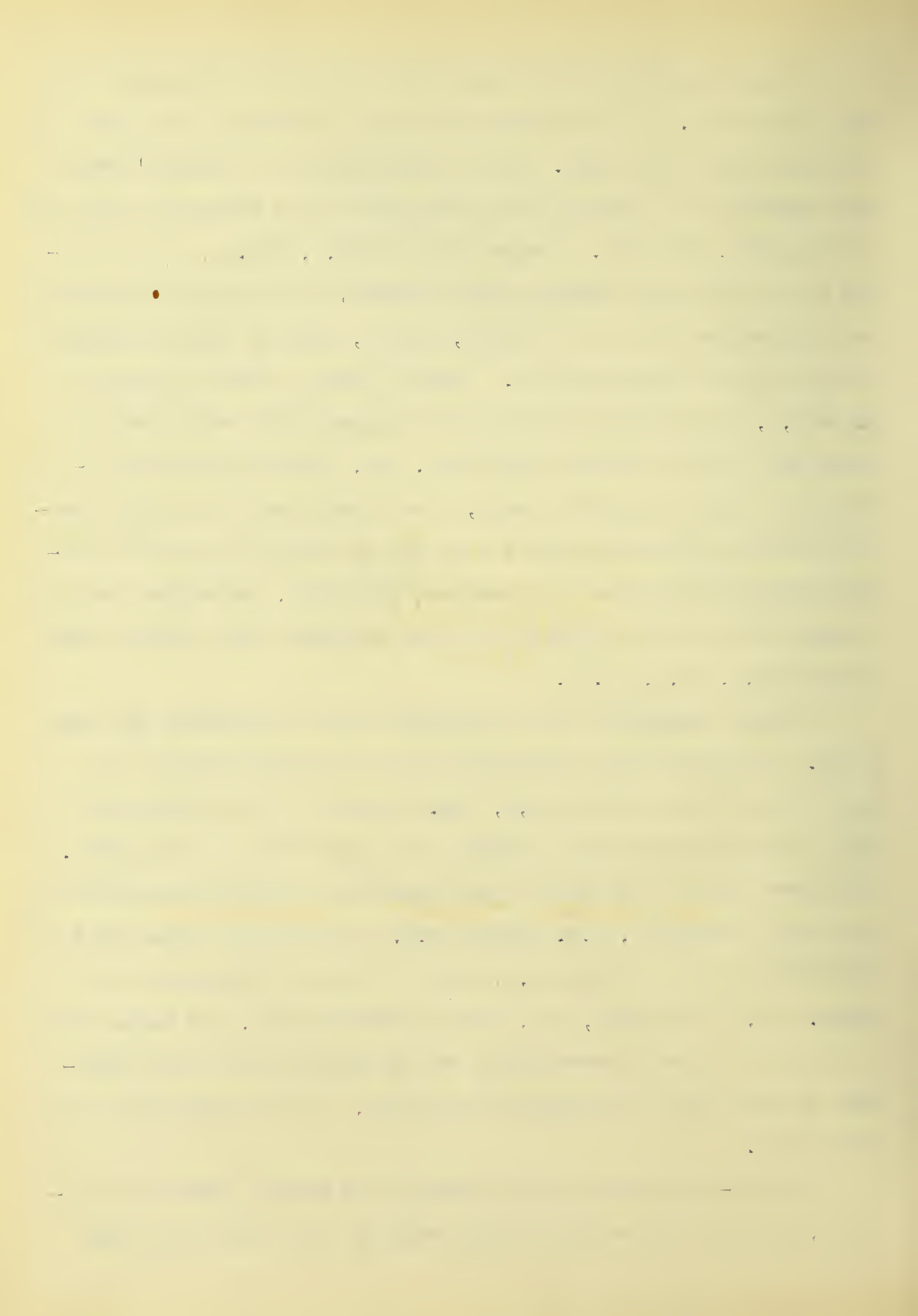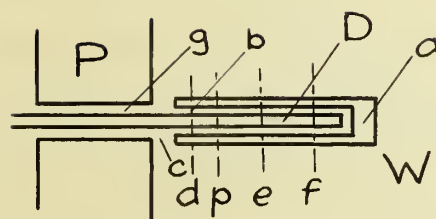
SECONDARY MAGNETIC CIRCUIT. (Fig. 13).

with a brass one. The results are shown in Fig. 15. It is seen from these curves that the presence of iron at,a, increases the torque effect some fifteen per cent in this case, thus establishing a check on the first proof.

Torque Effect. The force exerted on current in a field is obviously proportional to the product of the two. In determining the usefulness of this pull or the effective torque, it is necessary, however, to consider the direction of the pull. This pull will be in a direction perpendicular to the direction of the current. Now the effective pull is that exerted in a direction perpendicular to the radius through, O . It is evident that certain elements as at,2, Fig.16 will contribute little effective torque as the pull is almost radial, while at other elements as at,1, the pull is perpendicular to the radius and is consequently effective.*

There are two other factors which enter into the determination of the effective pull besides the direction of the current,i.e. the flux and the radius of the point. It was concluded that the maximum flux cut the disc at a point close to the pole, considering this and also the fact that the moment is proportional to the radius, it is to be inferred that elements such as at,1, are certainly the most effective.

Instantaneous
 Values
of Torque.   The torque of the meter has previously been
                 treated only by its effective values; it is evi-

*The effective pull of any element is proportional to projection
  on radius.
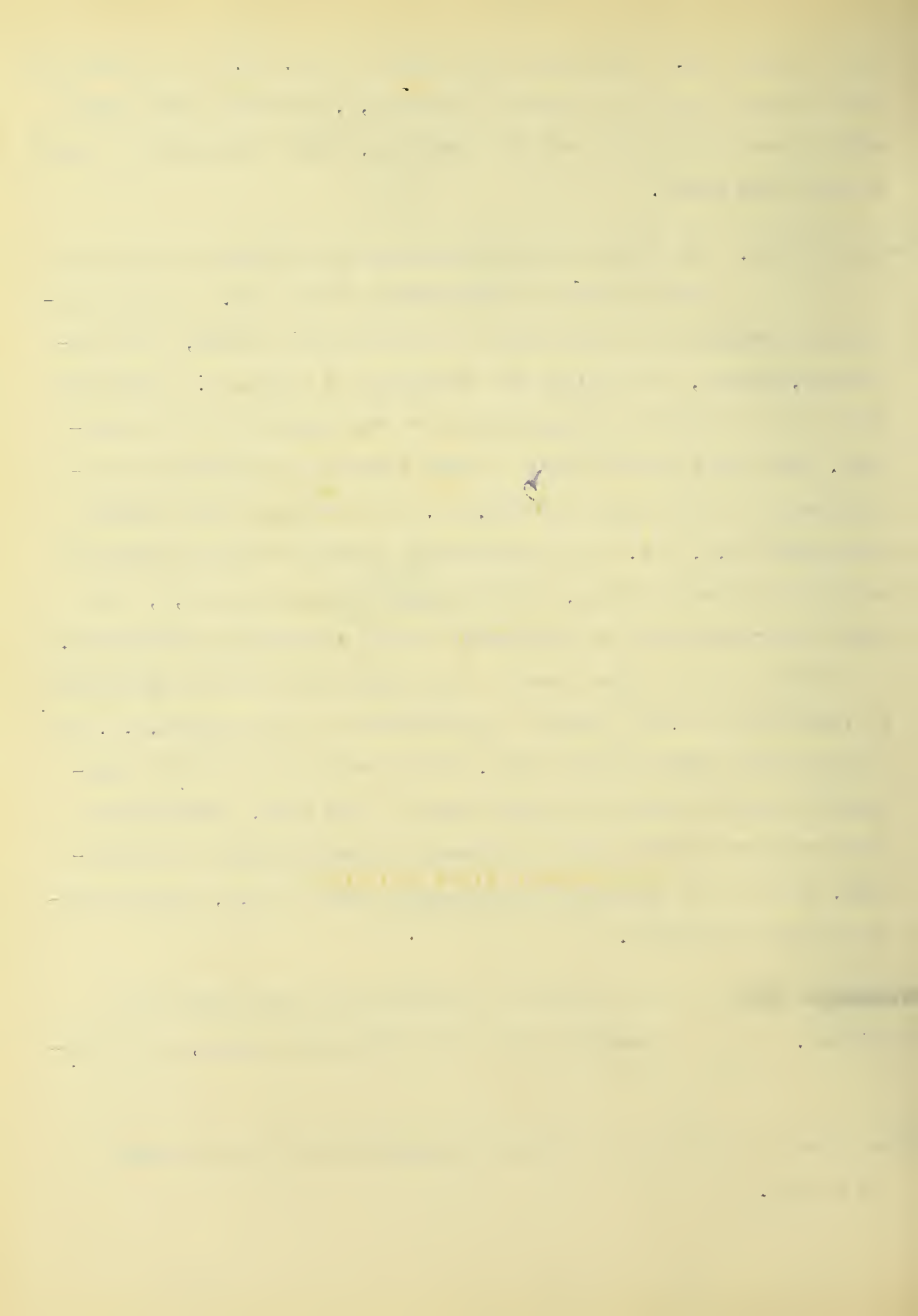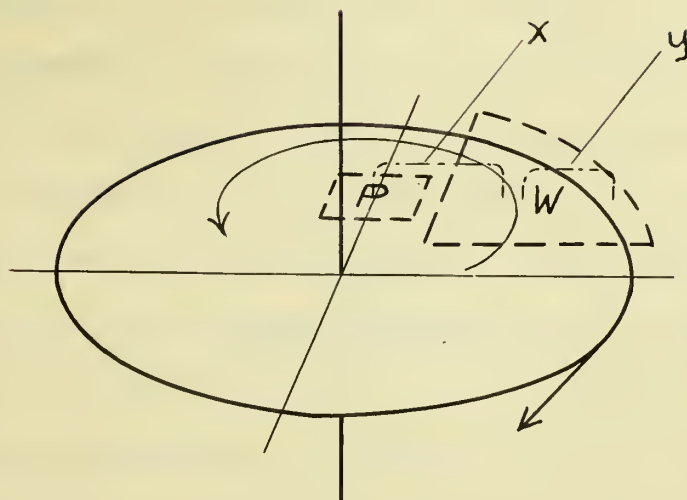
SECONDARY FLUX. (Fig.14).

dent,however, that from the alternating nature of the current and flux, the torque will not be constant but will vary from one instant to the next according to a cycle dependent on those of the current and flux, respectively. It will be convenient to assume sine wave forms of current and e.m.f.

If a circuit of impressed voltage,E, have a current,C,lagging by the phase angle ϕ .Fig.17, the winding of the upper element will have a current which is the vector sum,A, while in the lower one a current,B, corresponding to the vector difference will flow. This refers to the modified form of meter as Fig.6 in which the current and e.m.f. are combined in a single winding. It is obvious that the resultant flux and hence the effect is the same with either form.

It will be shown that the effective torque is independent of the angle between the vectors,A and B, and is proportional to the difference of their squares as previously stated. See p

Let,a, be the instantaneous value of the current in the upper coil and,b, that in the lower

$a = A \sin w\, t$

$b = B \sin(w\, t + \phi)$

$t_1 = K\, a^2 = K\, A^2 \sin^2 w\, t$

$t_2 = K\, b^2 = K\, B^2 \sin^2 (w\, t + \Theta)$

$t = t_1 - t_2 = K(a^2 - b^2) = K \left[A^2 \sin^2 w\, t - B^2 \sin^2 (w\, t + \phi) \right]$

$= K \left[A^2 \dfrac{1 - \cos 2w\, t}{2} - B^2 \dfrac{1 - \cos 2(w\, t + \Theta)}{2} \right]$

$= K \left[\dfrac{A^2}{2} - \dfrac{A^2 \cos 2w\, t}{2} - \dfrac{B^2}{2} + \dfrac{B^2 \cos 2(w\, t + \Theta)}{2} \right]$

$= K \left[\dfrac{A^2 - B^2}{2} - \dfrac{K}{2} \left[A^2 \cos 2w\, t - B^2 \cos 2(w\, t + \Theta) \right] \right]$

$= \qquad -\dfrac{K}{2} \left[A^2 \sin (2 w\, t + 90°) - B^2 \sin 2(w\, t + \Theta) + 90° \right]$
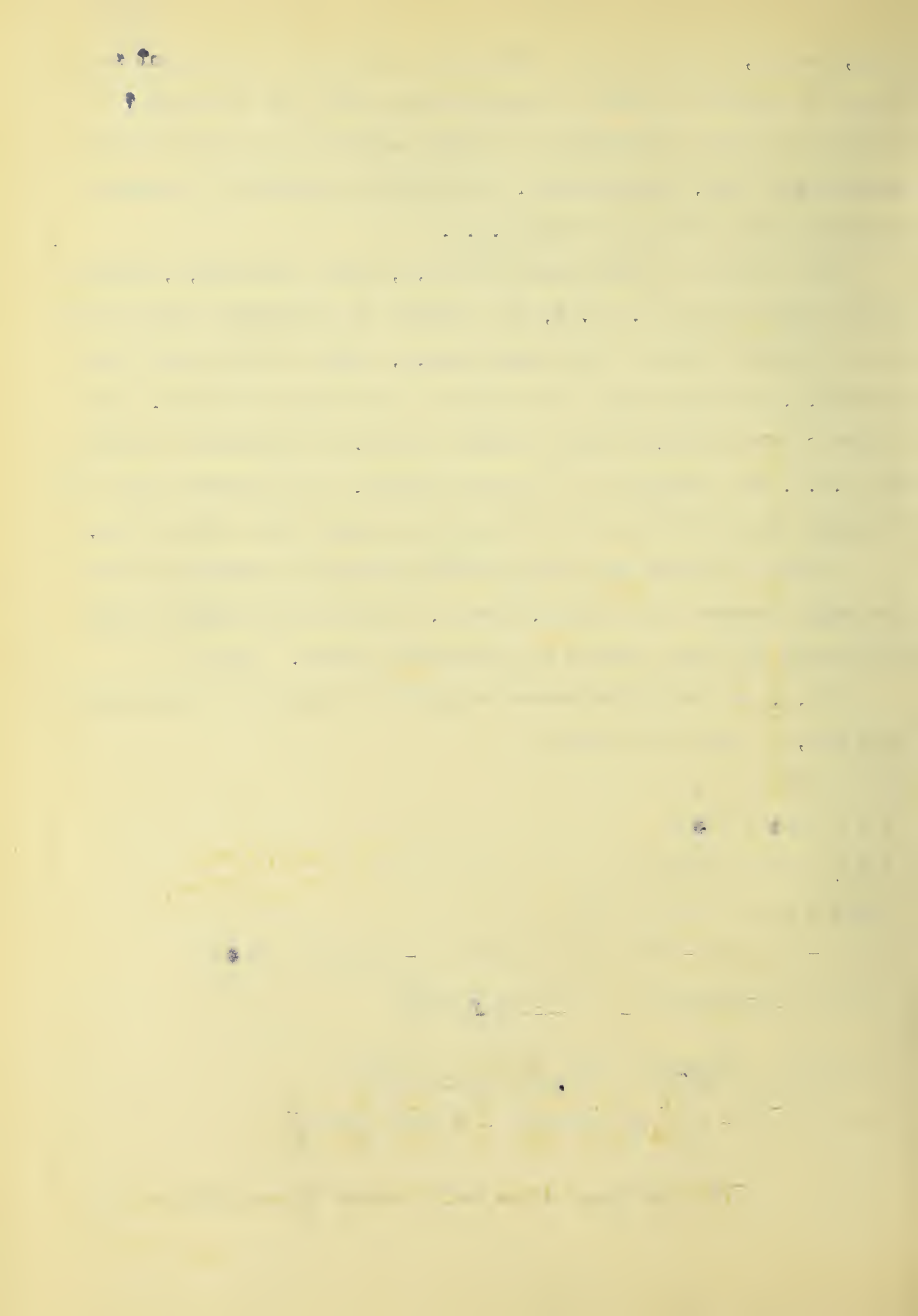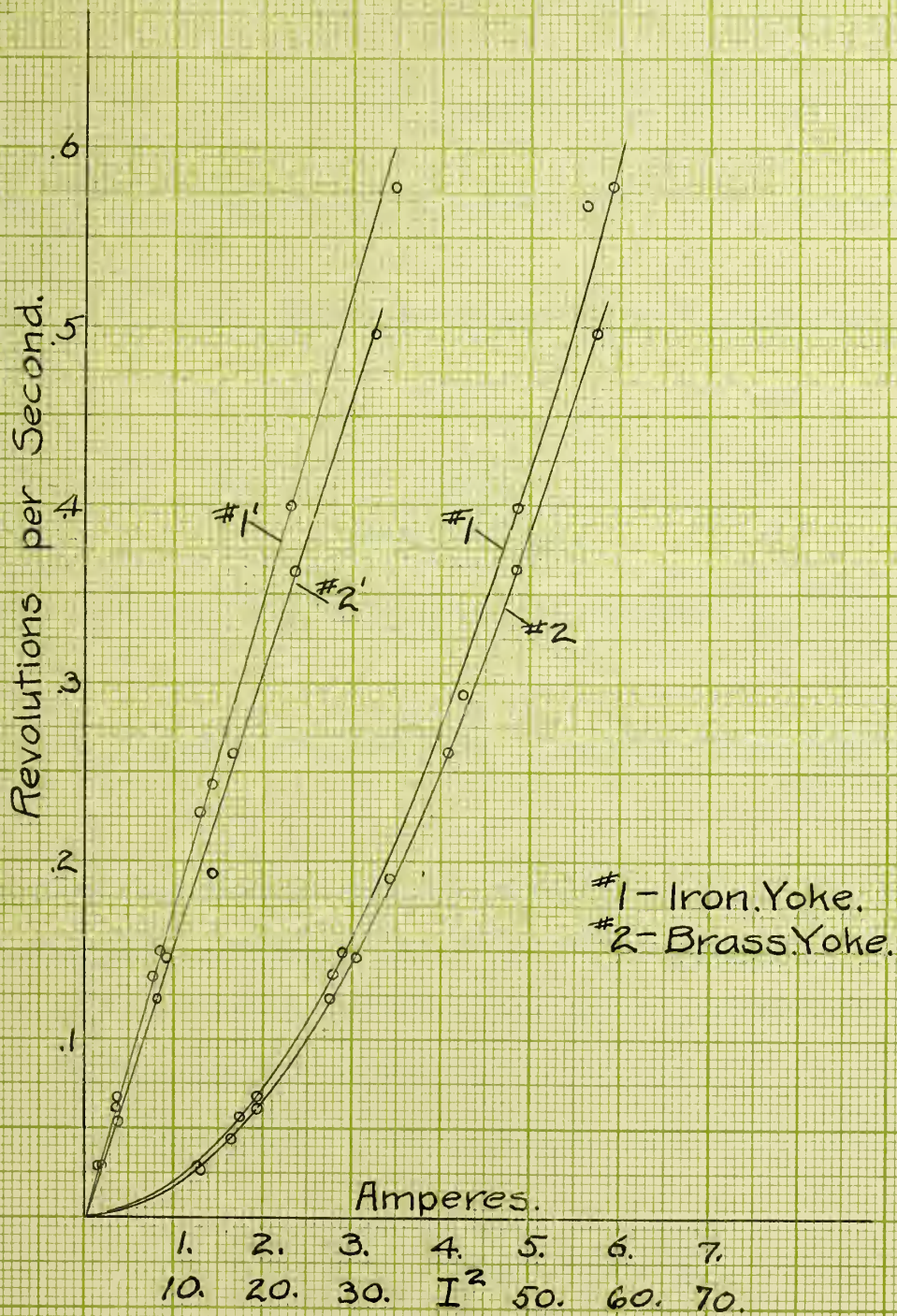
EFFECT OF IRON AND BRASS YOKES. (Fig. 15).

TEST #20.

$$= K\frac{A^2-B^2}{2} - \frac{K}{2}\left[A^2 \sin 2(wt+90°) + (-B^2)\sin(2wt+90°+2\theta)\right]$$

Let $\alpha = 2wt+90°$ & $\beta = 2\theta$

$$= K\frac{A^2-B^2}{2} - \frac{K}{2}\left[A^2\sin\alpha + (-B^2)\sin(\alpha+\beta)\right]$$

$$= \quad -\frac{K}{2}\left[A^2\sin\alpha + (-B^2)(\sin\alpha\cos\beta + \cos\alpha\sin\beta)\right]$$

$$= \quad -\frac{K}{2}\left[A^2\sin\alpha + (-B^2)\sin\alpha\cos\beta + (-B^2)\cos\alpha\sin\beta\right]$$

$$= \quad -\frac{K}{2}\left[(A^2+(-B^2)\cos\beta)\sin\alpha + (-B^2)\sin\beta\cos\alpha\right]$$

$$* = \quad -\frac{K}{2}\left[\sqrt{(A^2+(-B^2)\cos\beta)^2 + ((-B^2)\sin\beta)^2}\ \sin\left(\alpha+\tan^{-1}\frac{(-B^2)\sin\beta}{[A^2+(-B^2)\cos\beta]}\right)\right]$$

$$= \quad -\frac{K}{2}\left[\sqrt{A^4-2A^2B^2\cos 2\theta+B^4}\ \sin\left(2wt+90°+\tan^{-1}\frac{(-B^2)\sin 2\theta}{A^2-B^2)\cos\theta}\right)\right]$$

$$= \quad -\frac{K}{2}\left[\sqrt{A^4-2A^2B^2\cos 2\theta+B^4}\ \cos 2\left(wt-1/2\tan^{-1}\frac{\sin 2\theta}{\frac{A^2}{B^2}\cos 2\theta}\right)\right]$$

The final expression for the torque is seen to consist of a constant term and a harmonic of twice the fundamental or impressed frequency. These relations are shown graphically in Fig.17 in which are represented values of the currents, a and b, the corresponding values of the torque, T_1 and T_2 and their resultant, T.

*Note:- $P\sin x \quad Q\cos x = \sqrt{P^2+Q^2}\ \sin\left[x+\tan^{-1}\frac{Q}{P}\right]$
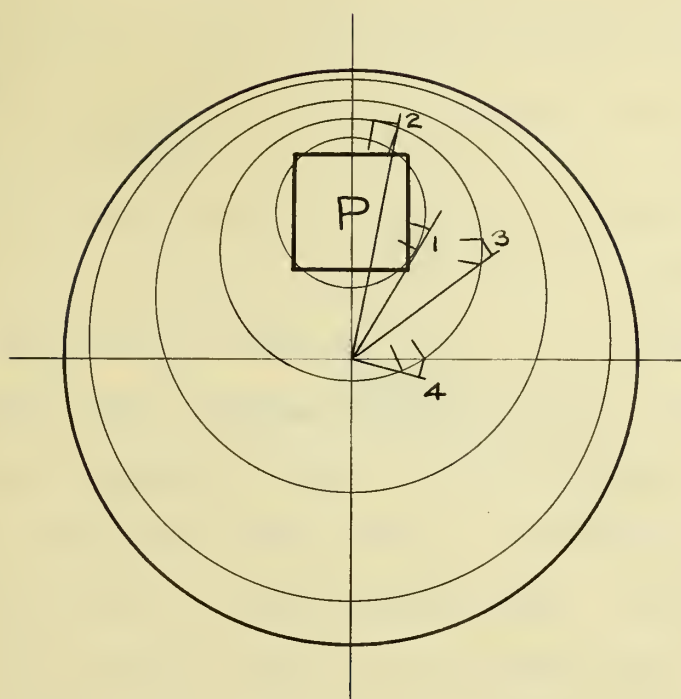
TORQUE EFFECT OF CURRENT. (Fig. 16).

It will be noted that the values of, T_1, are always negative,
it was plotted above the axis for convenience in constructing the
net torque curve, T, the instantaneous value of which is equal to
the difference of, T, and T_2 at any instant.

The rather complex expression obtained is of the form

$$T = N + P \cos 2 X$$

where T is the value of the net torque at any instant. N is the
constant term, $K \dfrac{A^2 - B^2}{2}$, P the coefficient of the harmonic func-
tion. It is evident that for any number of complete cycles the
second term will go through an equal number of positive and nega-
tive values, so that the average value of the torque will not be
effected by this term. This is shown in Fig.18. The shaded area,
R,represents the negative values of the torque, T, which occur
between a and b and the second area, S, represents an equal portion
of the positive values. The remaining area, represents the use-
ful torque. The average value of the torque for a cycle will be
this area divided by its base or the length of the cycle. The mean
value is the line, m n, whihc is at a distance N from the axis.
This line is the axis of the harmonic.

The maximum value of the torque will occur when $\cos 2X = 1$,
when $M = N + P$. This maximum value is probably effective in set-
ting up vibrations in the disc.

The mean value of the torque and that which is effective in
producing rotation of the disc will be the average value $N =$

$K \dfrac{A^2 - B^2}{2}$, which is independent of the angle between A and B.
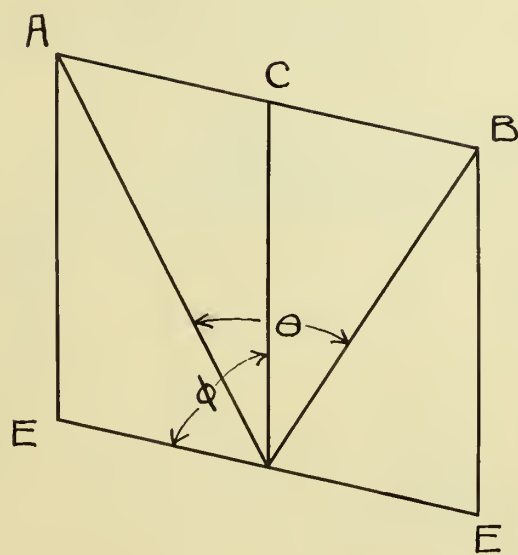
VECTOR RELATIONS. (Fig. 17.)
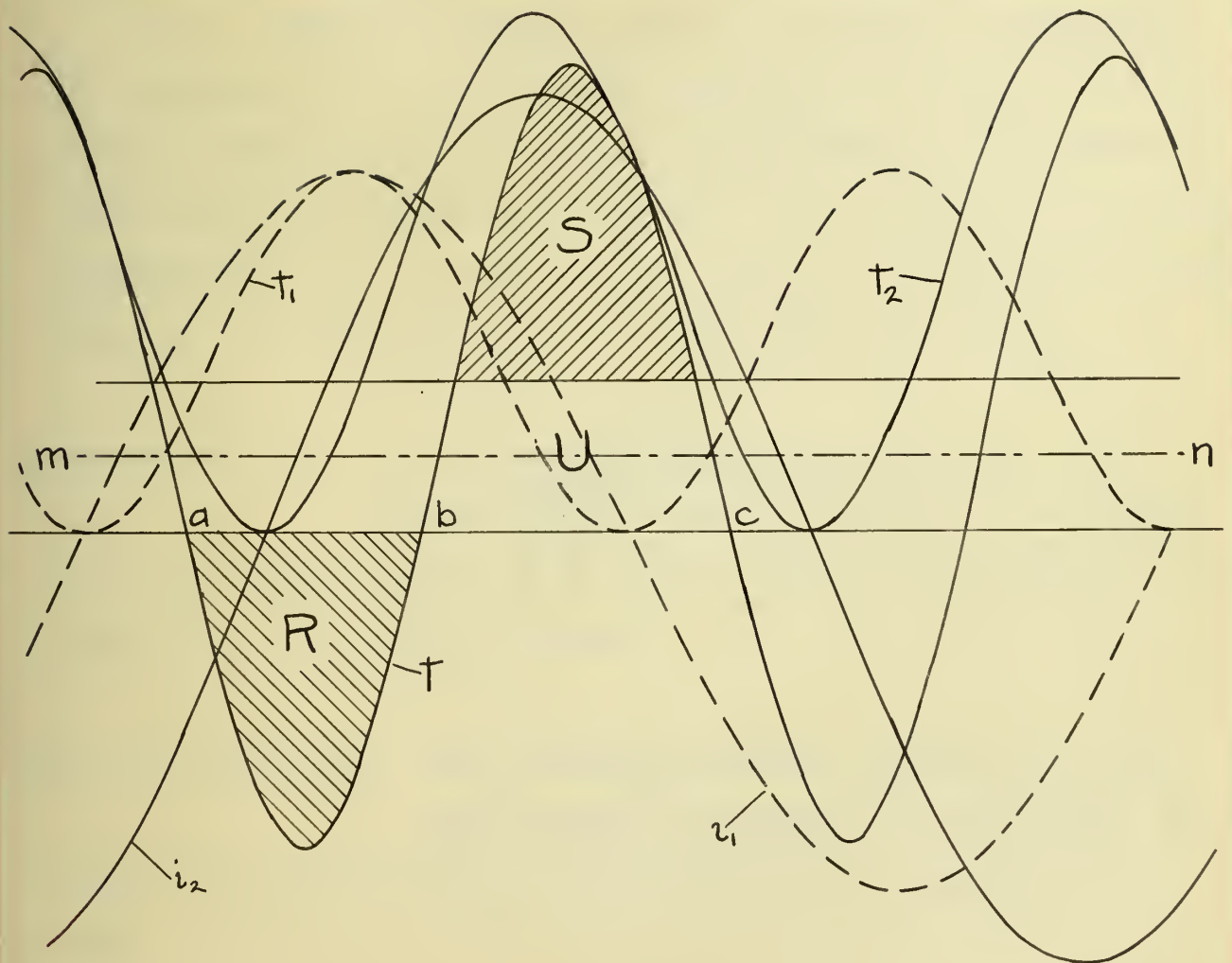
PULSATING CHARACTER OF TORQUE. (Fig. 18.)

4. <u>Experimental</u> <u>Investigation</u>.

It is the purpose of the following work, most of which is experimental; first, to verify the fundamental laws upon which the theory is based and second, to discover, if possible, the secondary phenomena or reactions which accompany the former.

These secondary reactions which it is safe to assume, exist in almost any piece of electrical apparatus to a greater or less extent, become of relatively great importance in an apparatus intended for such purpose as the present where accuracy is the first consideration. It is here that the value of the fundamental principle and the limitations of its application may depend.

Fundamental Law. The fundamental characteristic of the meter element has been called the "law of the meter". It may be stated simply, as proportionality of the torque to the square of. the energizing current.

Secondary Phenomena. The secondary phenomena include all other actions which accompany the fundamental or occur in the application of the fundamental to the purpose in hand.

The principle secondary phenomena are treated under the following heads:-

Frequency   (b)Power Factor.

Secondary Reaction

Friction

Self-braking Effect

Transformer

(a) Ratio

The necessity of moving parts in any mechanical device brings in the element of friction. This factor is one of the most difficult to treat and notwithstanding its wide importance in the meter field no practical means have as yet been found which will eliminate this harmful effect.

In machines of the present type where a hardened pivot turns on a jeweled bearing, the effect is minimized to a great degree but under the best of circumstances it may be found to exist. Its presence is so universal that it will be found necessary to consider its effect in each individual case.

In the above list the first two items are characteristic of the device; the first having been treated in the earlier theory. The self-breaking effect enters in the application of the revolving disc and the last items are brought in by the use of the transformer as a means of providing the e.m.f. factor to the metering device.

The self-braking effect will be mentioned, here, as it enters most directly into the consideration of certain experiments relative to the fundamental law. This fundamental law is possible of direct proof by a measurement of the quantities concerned i.e. torque and current. It is convenient,however, to further demonstrate this this law in another manner and by a principle which is quite widely employed in current meter practice.

If the rotating element is equipped with a retarding device the drag of which is proportional to the speed, the speed will in turn be proportional to the torque. It is in this latter relation that the self-braking effect enters. However, it will be

TORQUE BALANCE. (Fig. 19).

shown that within certain limits the "effect" is negligible and
we have a linear relation of the torque and speed. This leads
us to a second and more directly applied law ⊨ the proportionality
of the speed of the disc to the square of the current.

Fundamental Law.

Torque Current Curves. The meter has been described as a com-
bination of two motor elements working
in differential. Each of these elements
consists of an individual core, disc and wing and is complete in
itself.

In the investigation of the "law of the meter" one of these
elements was used separately being entirely disconnected from the
other except through the common shaft upon which both discs were
mounted.

In these tests the method of procedure was a direct measure-
ment of the torque by means of a torque balance. The balance, as
shown in Fig.19 consists of bell crank lever made of aluminum
wire and pivoted on a steel needle. This lever transmits the
force exerted by the disc to the weighing system which consists
of a Jolly balance.

The operation consisted in balancing the pull of the rotat-
ing element against the stretch of the spring and then calibrating
the spring by known weights placed in the pan. Then from the ra-
tio of the lever arms the torque could be reduced directly to
gram centimeters.

The index of the balance being adjusted to zero and the

reading observed while the circuit was open, the current is brought to the desired value and the index again adjusted to zero and a seconding reading taken. Similar readings being taken for values of current throughout the range. The results of such tests of which several were taken at different frequencies are shown in Fig. 20, 21, 22.

These curves are seen to be of the form of the parabola; the general equation being $y+a = bx^2$ If in this latter expression, z be substituted for, x^2 then, $y+a = bz$. which is the equation of a straight line. If the ordinates of the curve, I, be again plotted using the square of the abscissae of, I, as the new abscissae, we obtain the curve ,II. This curve is seen to be a straight line as anticipated above. This curve is quite useful in the considera-tion of the first curve; for if the second is a straight line then the first must be a parabola; it being much easier to discover any variation from the straight line than an equal variation from the more complex curve.

Owing to the difficulty of accurate determinations for low values of the current the torque current curve was not drawn to the origin. We may however make some inference as to the lower part of the curve by using the "test" curve mentioned above.

The curve, II, was found to be a straight line for a considera-ble range of current, deviating somewhat for the higher value of the current. If we produce this curve it does not pass through the origin as we might at first expect but is seen to cut the vertical axis somewhat below the origin. It follows that curve, I, must also pass through this same point. We may then extend our torque curve to this point.
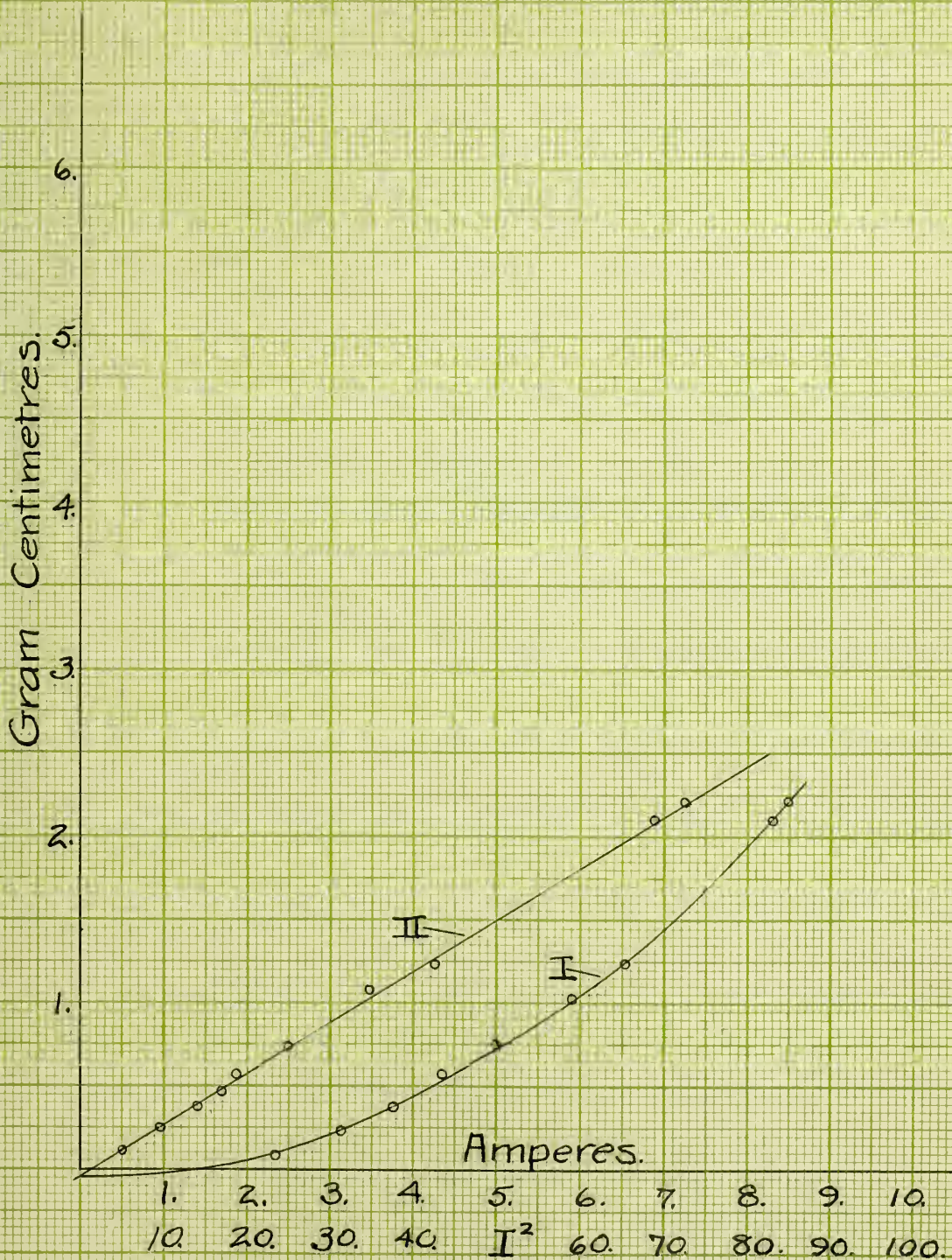
TORQUE-CURRENT CURVE ─ 30 ν. (Fig 20).
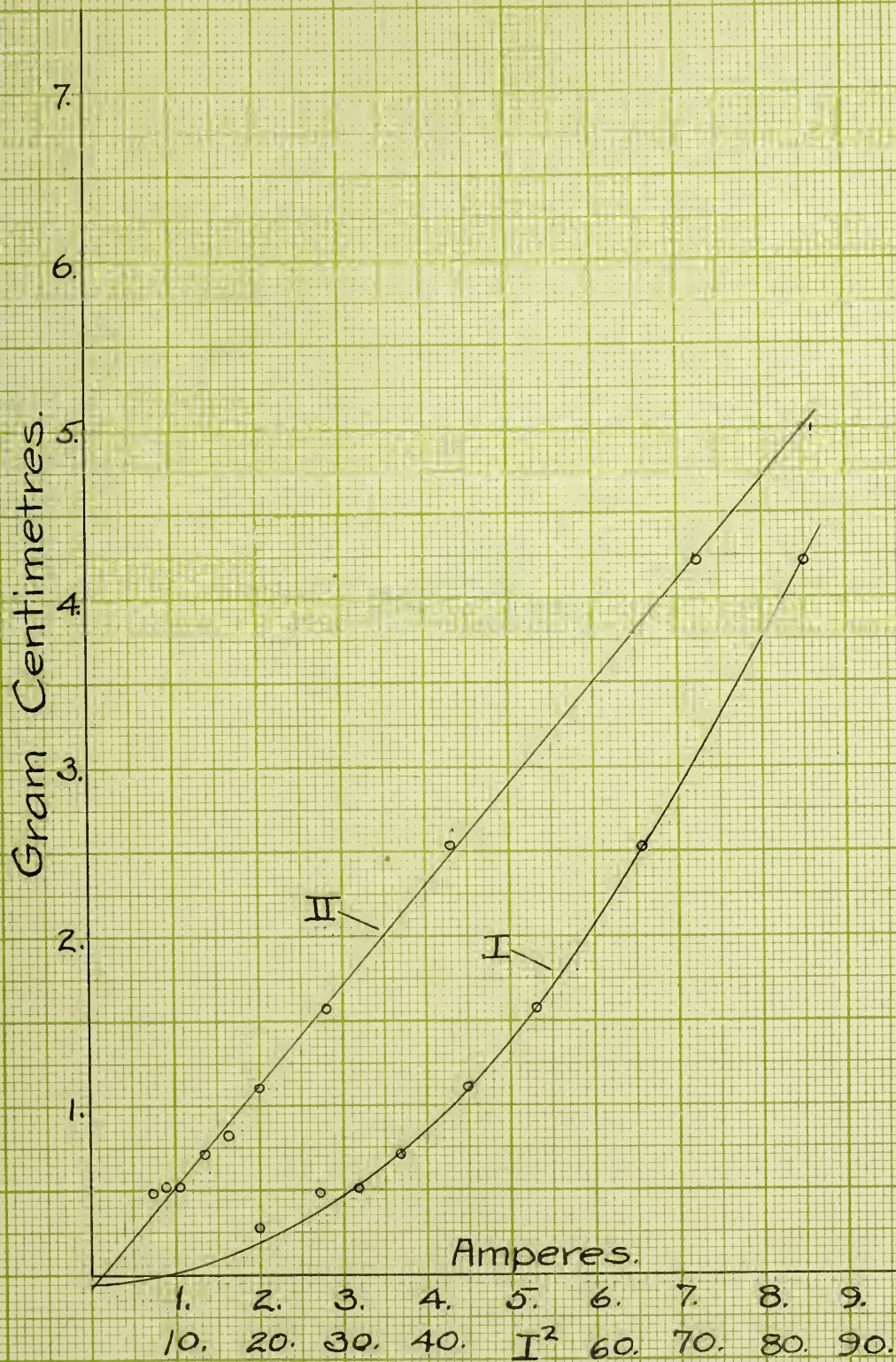
TEST #11a.

TORQUE CURRENT CURVE -45∿.(Fig.21).

TEST #71c.
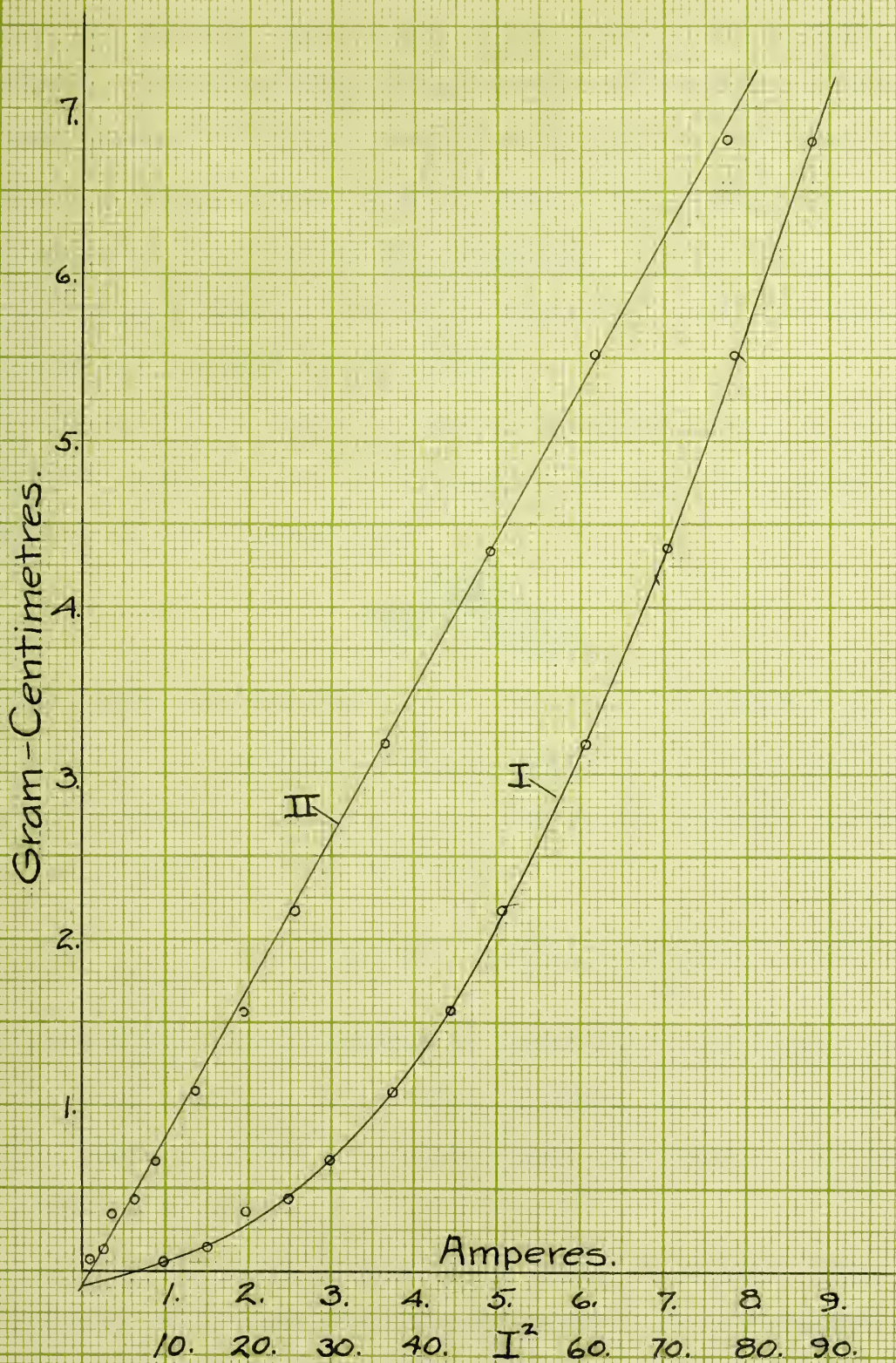
TORQUE – CURRENT CURVE – 60∿. (Fig. 22).

TEST #14-a.

It is now seen that the torque current curve cuts the horizontal axis at a point to the right of the origin. This intercept represents the current below which there would be no torque exerted at the shaft. The intercept on the torque axis which is negative, corresponds to the "negative torque" which must be overcome before there is any tendency of the disc to move. This force corresponds to the mechanical friction of the bearings. If our curve,II, were exactly a straight line, it is to be concluded that the friction is constant and independent of the torque.

The effect of the friction can be shown in a more accurate way in another manner. If the torque of the disc were proportional to the square of the current then

$$\frac{T}{I_2} = K$$

Fig.23 represents the data of Fig.   plotted in this manner. The curve is seen to be decidedly concave towards the axis and the value of,K. is not constant throughout the range. If,however, as indicated by the previous curves the form

$$T + a = KI^2 \quad \& \quad K = \frac{T+a}{I_2}$$

be used where,a, is the intercept as taken from the torque current curve it is seen that the value of,K, becomes very nearly constant. This again leads us to the conclusion that the friction is practically constant in value.

From a consideration of the above facts it appears that the torque produced by the current is proportional to the square of the current but owing to the mechanical friction of the bearings the effective torque departs slightly from this law by an amount which is evidently proportional to the degree of mechanical friction
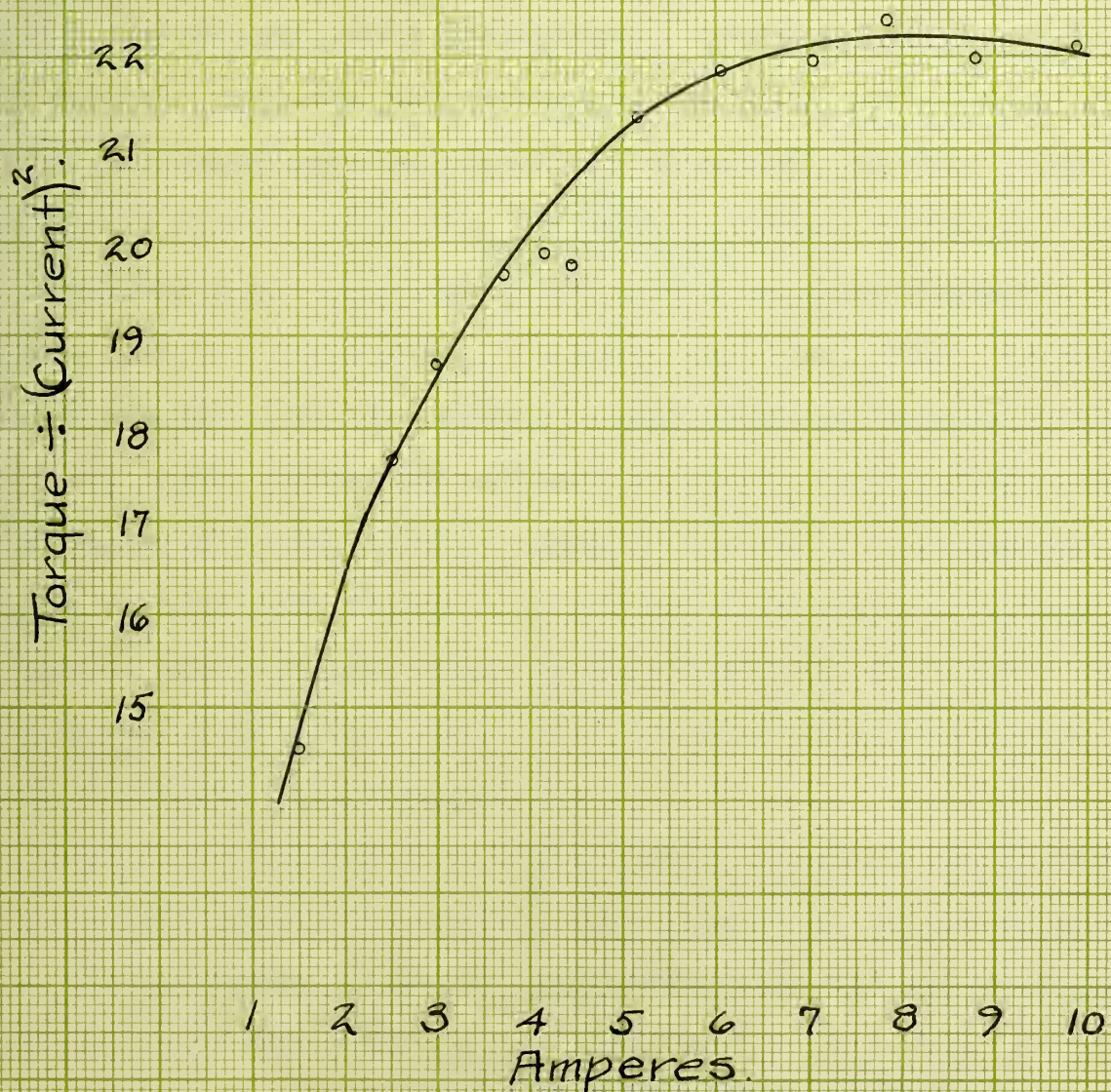
Motor Characteristic (Fig.23).

Test #14a.

present in the instrument.

Speed Current Curves. In these tests, the object of which is to determine the relation of the speed and energizing current, the same circuit was used as in the previous tests, i.e. the lower element. The upper disc being equipped with drag magnets furnishes a suitable retardation without effecting in any manner the action of the motor element.

The procedure here consists of observing the speed of the discs for various values of the current. The speed of the disc being computed from the revolutions of the disc, as counted, and the time corresponding, in seconds as obtained by a stop watch. These observations being taken for various values of the current.

The results of such tests are plotted in Figs. 24,25,26.27, taken at various frequencies. In these tests as before the test curves are drawn using the square of the observed current as the abscissae. Both curves are similar in general form to those obtained in the previous tests. There is however one difference that has already been referred to and which is quite noticable,i.e. the drop of the "straight line" or test curve,II, for higher values of the speed. This effect which will be taken up later is seen to be negligible at the lower speeds.

These curves produced as before again intercept the vertical axis at a point below the origin. This intercept again corresponding to the mechanical friction, indicating that the curve of the instrument is of the form

$$\frac{r}{s} + a = KI^2, \text{ or } K = \frac{\frac{r}{s} + a}{I^2}$$
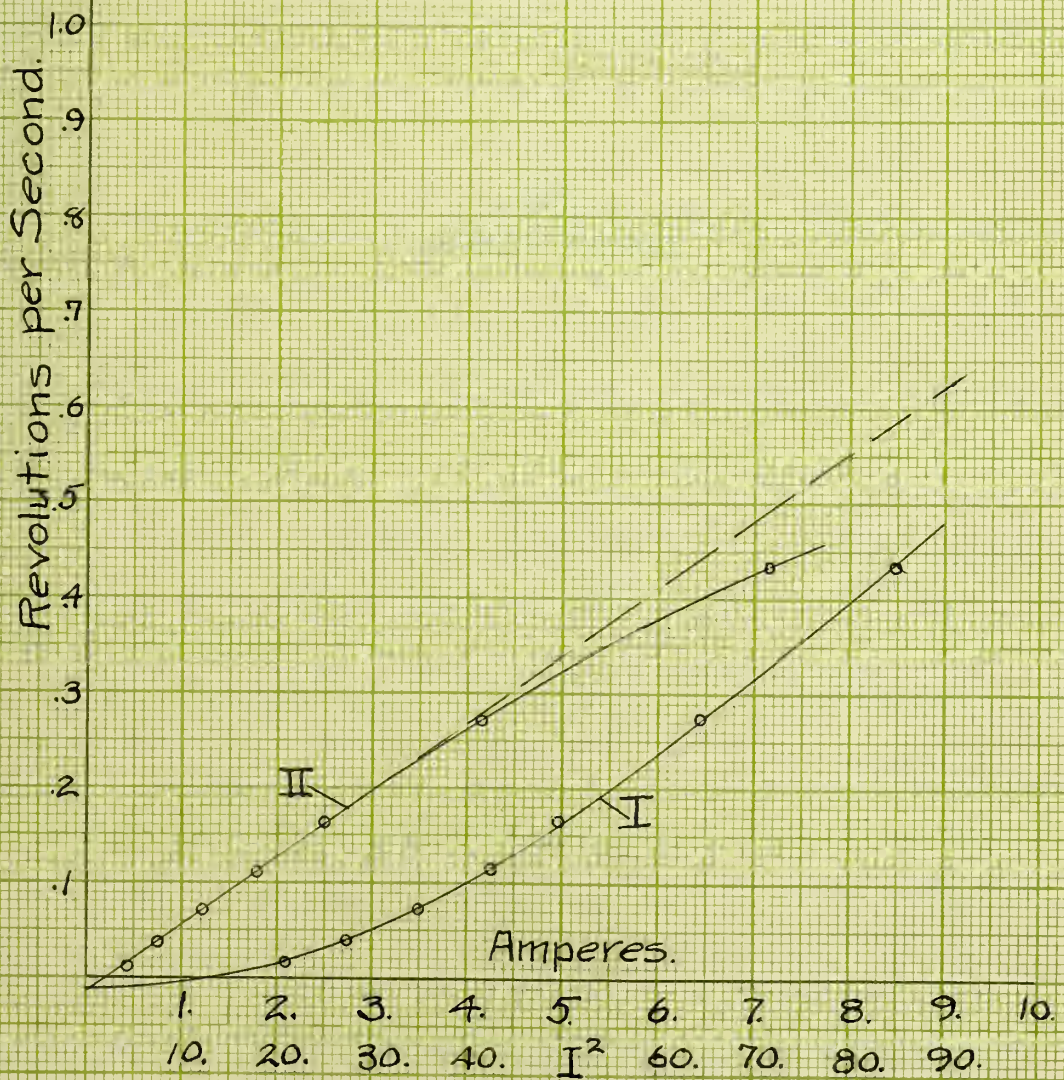
SPEED CURRENT CURVE – 30∿. (Fig. 24).
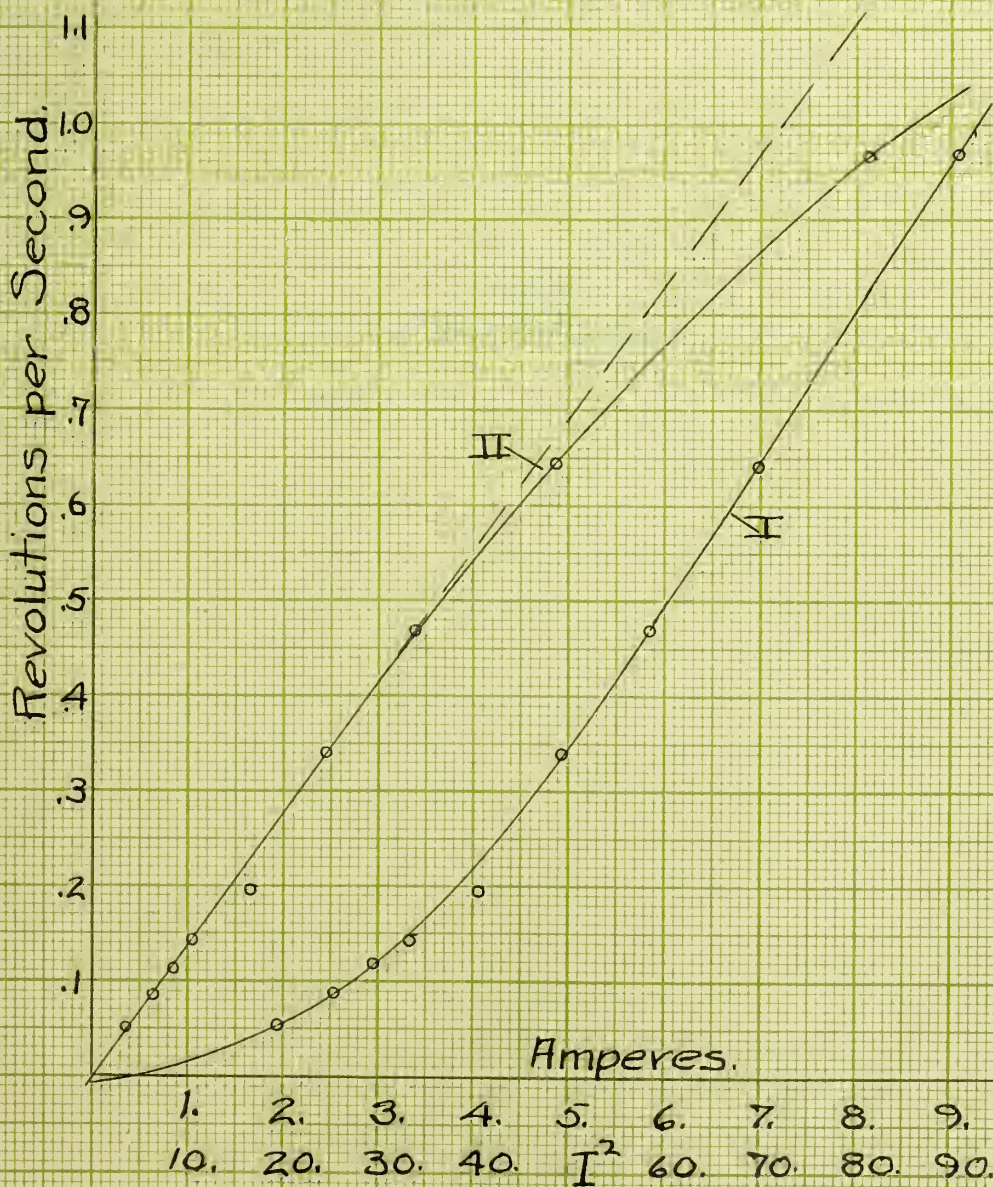
TEST #11 b.

SPEED CURRENT CURVE – 45∿ (Fig. 25).

TEST #11d.
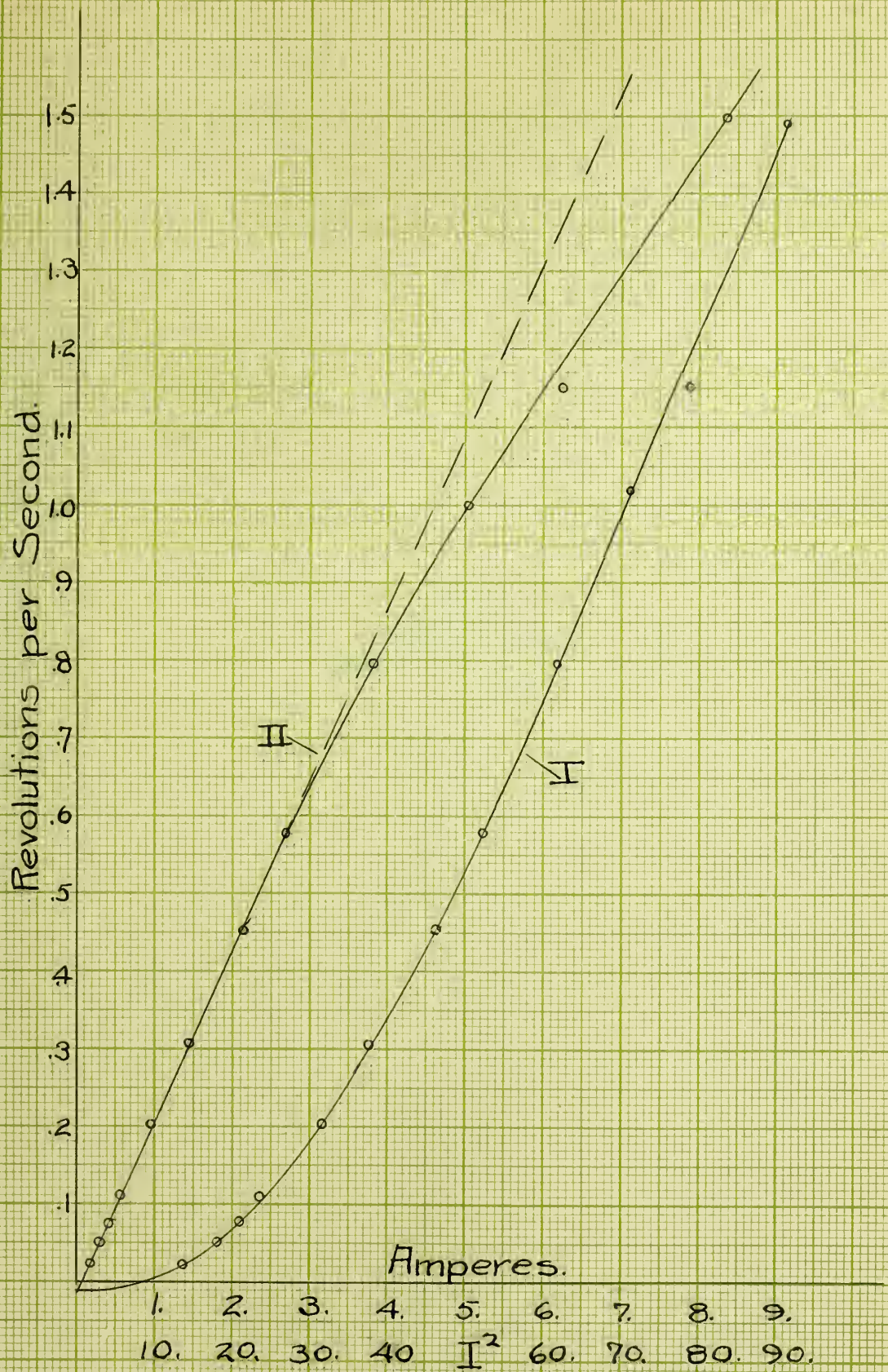
SPEED CURRENT CURVE 60∿ (Fig. 26).

TEST # 14b.

EUGENE DIETZGEN CO., CHICAGO.
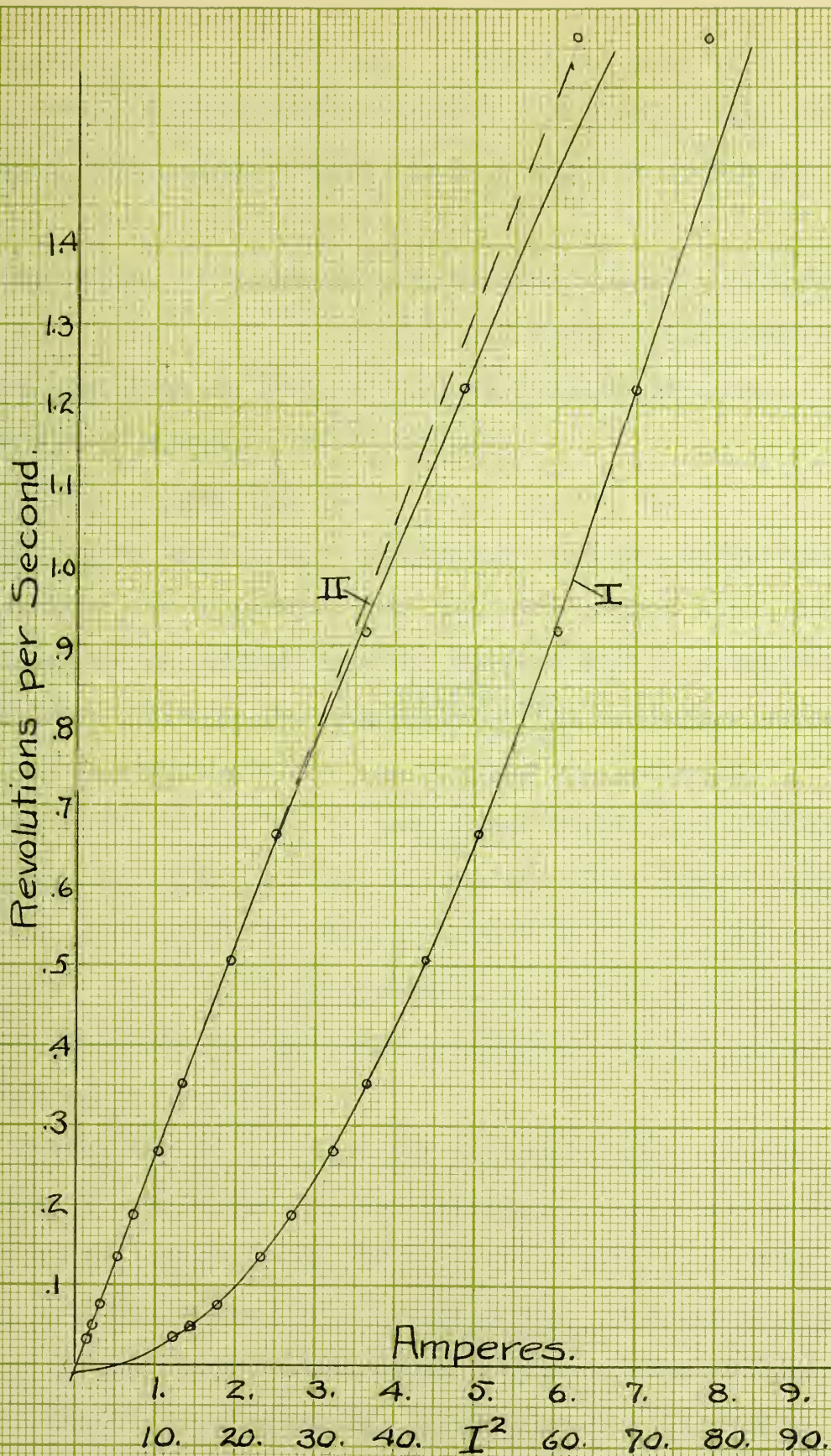
Speed Current – 71.4~. (Fig.27).

Test #12 d.

where, a, represents the friction. The form of the curves is seen
to bear out the proportionality of speed to current squared to a
close degree, throughout a long range of current frequency.

Secondary Phenomena. Frequency. The meter element is effected by
the frequency of the supply cur-
rents in a way which is shown by the expression derived for the
secondary e.m.f. where it was shown that

$$e_2 = K w I$$

and as the torque varies as the square of the secondary current,
it follows that the frequency will therefore enter into the second
power. There is one factor however which tends to a departure
from this law. This is the reactance of the secondary circuit
which tends to compensate for the changes in frequency.

Curves plotted between frequency and torque and also frequen-
cy and speed are shown in Figs. 28 and 29. These curves were plot-
ted from the curves of the torque current and speed current tests,
respectively. Each curve representing the value of speed or
torque corresponding to the current indicated on the tests taken
at the several frequencies.

The curves appear to be nearly straight lines between the
limits taken but intercepting the horizontal axis considerably to
the right of the origin. Fig.30 is a curve similar to Fig.29 but
extending over a greater range of frequency. The data being ta-
ken mostly at the lower frequencies. The curve then assumes the
well known form of the parabola. The previous curves, which were
apparently straight lines, being arcs of the same curve at a dis-
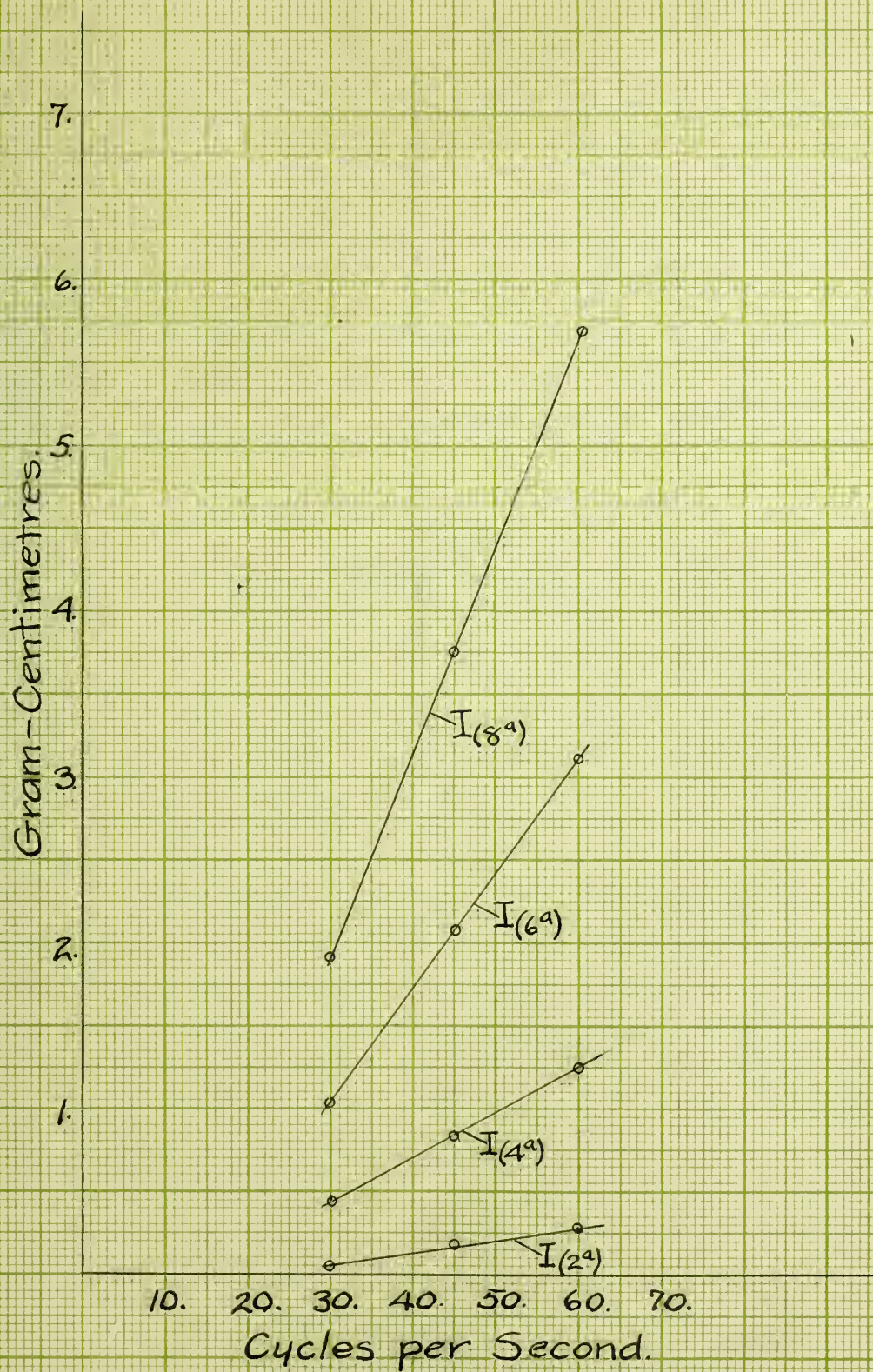
TORQUE FREQUENCY CURVE. (Fig. 28.)
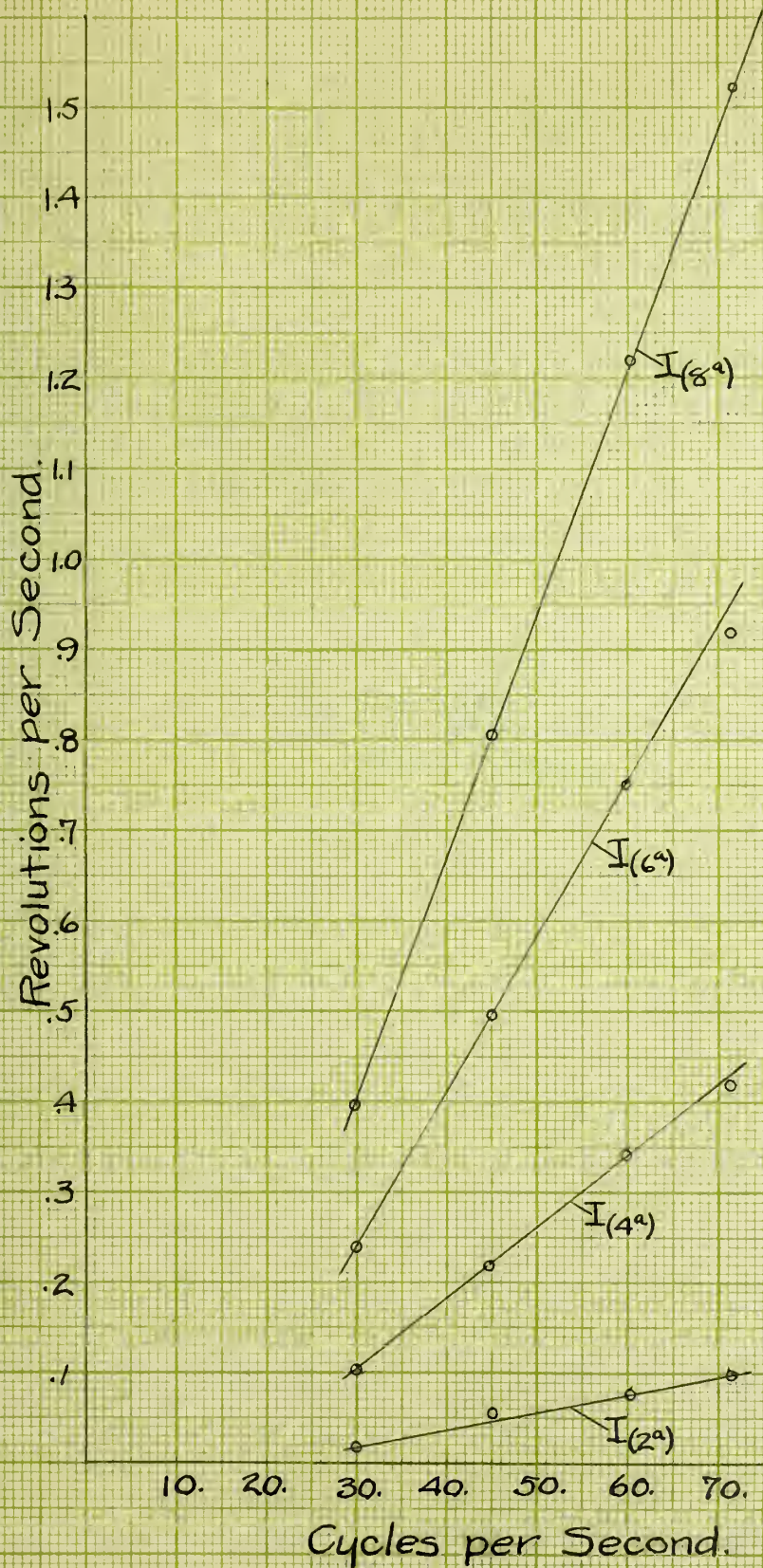
SPEED FREQUENCY CURVES. (Fig. 29).
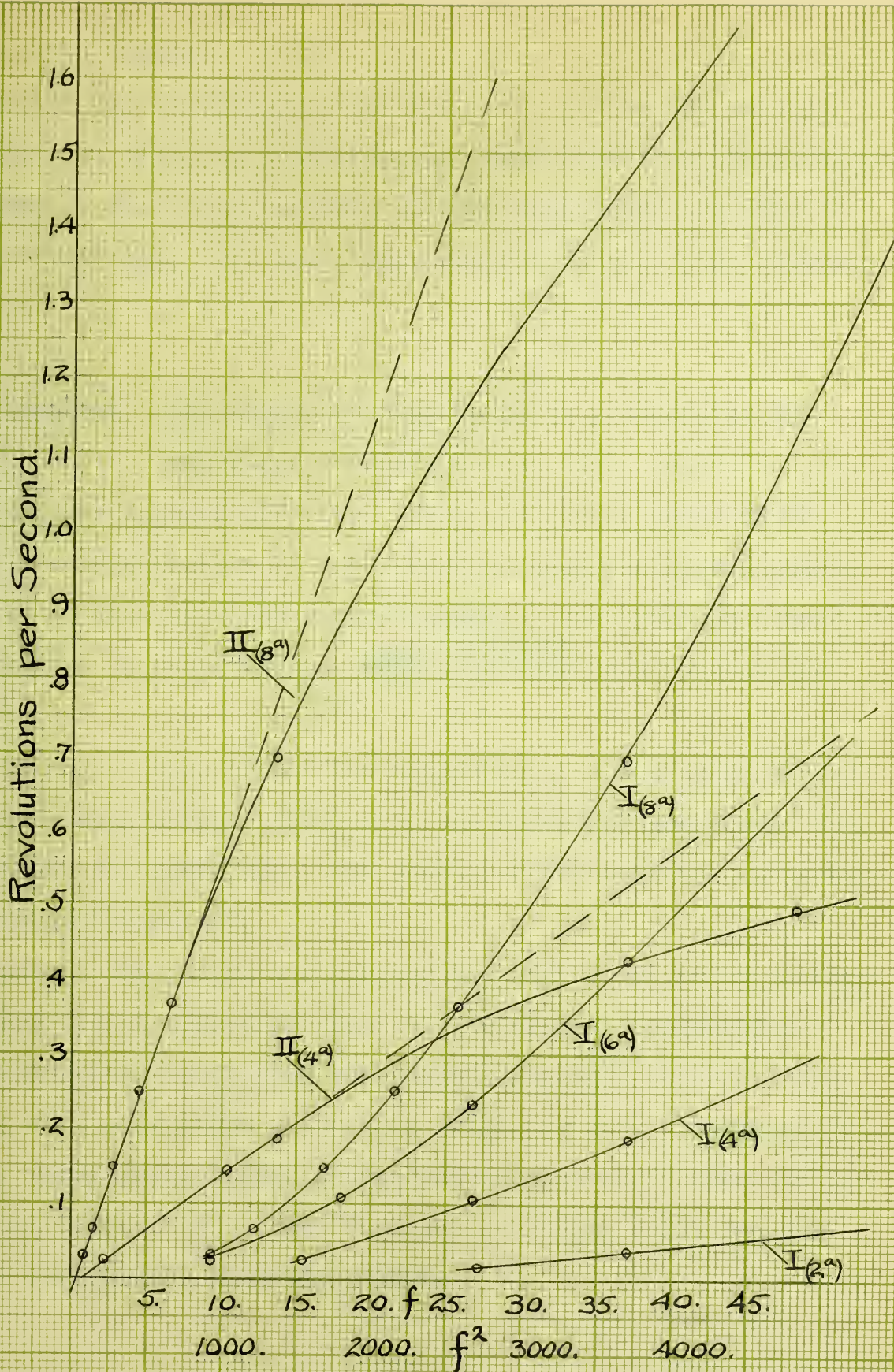
SPEED FREQUENCY CURVE. (Fig. 30).

TEST #23.

tance from the origin.

In Fig.30 the"test" curves are drawn for two of the curves.
It is seen that there is a considerable departure from the straight
line. This is due to two effects already mentioned; the first be-
ing the self-braking and the second the reactance of the secondary
circuit.

Secondary Reaction. The secondary flux which is produced by the
                          secondary currents in the disc, reacts to some
extent upon the primary circuit. This reaction is quite small
owing to the construction of the secondary magnetic circuit which
leads most of this flux away from the core. The currents complete-
ly surrounding the pole however send some flux back through the
core.

To determine this effect the reactance of the primary circuit
was measured with current flowing in the secondary and again with
no current in the secondary. The effect of the wing on the primary
core was also determined.

A current was passed through the winding of the meter element
and the drop was measured by means of a sensitive dynamometer.
Observations being taken when the element was equipped with disc
and wing; with wing alone; with disc alone and with neither disc
nor wing. From these drops as observed and the resistance the re-
actance was computed as below. Observations being taken for two
values of the current in motor winding.

Summary.

	4.20ª (approx)		5.87ª (approx)	
		X		X
Wing and disc	.1300	.1297	.1315	.1312
Wing alone	.1376	.1372	.1395	.1390
Neither	.1277	.1275	.1300	.1298
Disc alone	.1212	.1211	.1227	.1225

Resistance = .0232

Frequency = 60

From these results it appears that the impedance is greater in every case, as measured at the larger current. Further it is seen that the drop is almost entirely a reactive one, the resistance being quite small in comparison.

The reactance of the coil when equipped with neither wing nor disc was computed as .1275 (at 4.2ª). The addition of the wing increased this reactance to .1372 or about 8%. This means that this amount of flux was carried by the wing. As the disc was put in place the reactance again decreased, to .1300 or about 5.3%. This means that the secondary currents send a back flux of that amount which links with the primary. Another measurement was taken of the drop with the disc alone. This value shows an increase of only 5.1% over that without. A part of the difference between this and the first observation (5.3%) is probably due to greater reluctance of magnetic circuit due to absence of wing.

The effect of these reactions which as stated are very small, does not effect the operation of the instrument to any extent, as

these reactions tend only to combine with the primary flux to
produce a new flux which is again proportional to the current but
of a slightly reduced magnitude. Although the effect of the wing
is to increase the induction it is probable that the effect is
also harmful as the increased flux threads the gap of the wing
in a direction opposite to that of the useful flux.

Self-Braking Effect.   The retardation due to the action of a flux
on the current generated by it in a moving disc is a property
much utilized in current meter practice.   This phenomena is also
present in another manner besides that regularly employed as the
retarding device and constitutes a chief factor in the difference
between t he torque and speed curves.   It prevents the simple pro-
portionality of speed and torque from obtaining at higher speeds
it is of greater import than the element of friction.

The alternating flux of the core induces in the disc eddy
currents upon which the action of the device depends. There are
also currents generated by the revolution of the disc through this
flux. It is these currents in their action upon the main flux that
produce the self-braking effect.

These currents are alternating but as they are practically
in phase with the flux, their effect is a net torque opposing the
rotation. As it arises from the rotation of the disc through the
main flux, it has been called the self-braking effect.

If the disc rotate with a velocity, V, in a field of strength
ϕ_1, there will be generated an e.m.f. e'_2, where

$$e'_2 = a\phi_1 V$$

$$i'_2 = \frac{a\phi_1 V}{r_2}$$

$$T' = b i_2' \phi_1$$

$$T' = b \frac{a \phi_1 V}{V_2} \times \phi_1 = c \phi_1^2 \quad V$$

But $\quad \phi_1 = d I_1$

$$T' = c\, d^2 I_1^2 \, V = f\, I_1^2 V$$

$$V = g(T - T')$$

But $T = h\, I_1^2$

$$V = j(h\, I_1^2 - f\, I_1^2 V)$$

$$V = j\, h\, I_1^2 - jf\, I_1^2 V$$

$$V = \frac{j\, h\, I_1^2}{1 + j\, f\, I_1^2} = \frac{K\, I^2}{1 + m\, I_1^2}$$

$$\& \; T' = f\, I_1^2 \left(\frac{K\, I_1^2}{1 + m\, I_1^2} \right) = \frac{n\, I_1^4}{1 + m\, I_1^2}$$

Determination of Effect.   The two motor elements which comprise the differential system are connected only by the common shaft of the two discs. This combination affords a convenient method of determining the magnitude of the effect.

If the winding of a motor element be excited but the wing be removed it is evident there will be no rotational effort. If, however, the disc is rotated by some external force it will cut the flux of the excited core and the self-braking effect will be the same(neglecting the secondary flux) as if the wing were pressent and the disc rotated by virtue of its action. The effect depending on both speed and flux the disc must evidently be driven at the speed corresponding to the flux in the core under normal operation.

64
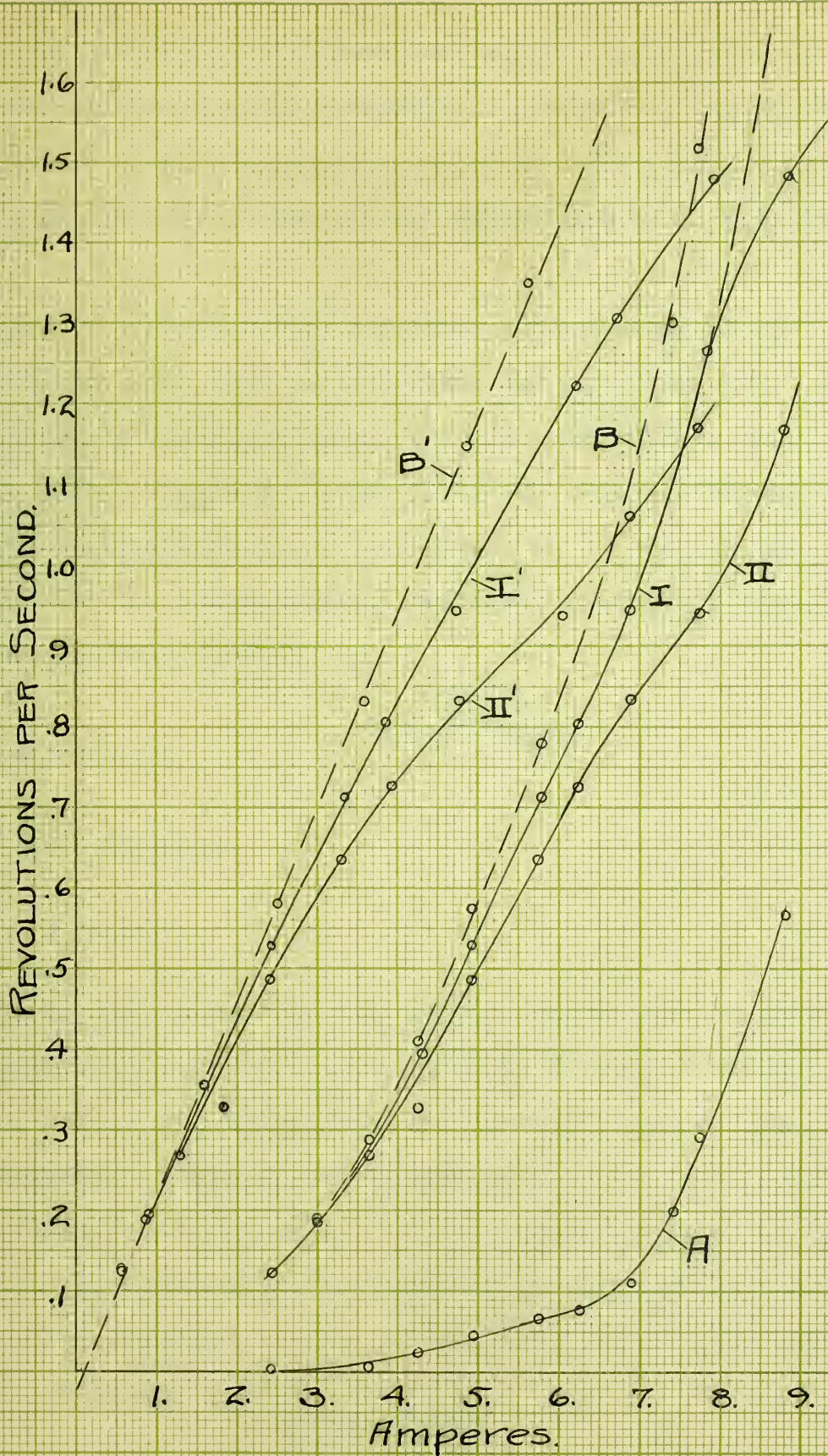
SELF-BRAKING EFFECT (Fig. 31).

TEST #18.

To accomplish this purpose the wing was removed from the upper core and the winding connected in series with that of the lower element which was equipped with its usual wing. The retarding magnets were placed on the upper disc as in previous tests.

If speed current curves be taken with the apparatus thus arranged it is seen that we will have not only the self braking effect due to the upper element but also that due to the lower i.e. the total effect is doubled. If the ordinary speed current curve be plotted on the same sheet it is obvious that the difference of the two ordinates for any current will represent the effect at that current, as

$$\left(\frac{r}{s}\right)_1 = V_1 = K(T-T')$$

$$\left(\frac{r}{s}\right)_2 = V_2 = K(T-2T')$$

$$\therefore V_1 - V_2 = T'$$

Fig. 31 represents the two curves. The points of the two being taken alternately at each current. Curve,I, is taken with the lower core,only, energized and Curve,II, being taken with both energized. Curve,A, was constructed by taking the difference of the ordinates of Curves,I & II, and Curve,B,was constructed by adding the ordinates of,A, to those of,I, which gives the curve that would be obtained if the self-braking effect were not present. This follows, as

$$(I) = K(T-T')$$

$$(A) = K T'$$

$$\therefore (B) = K T$$

As in the previous curves it is convenient to construct test curves plotting the speeds to the current squared as new abscissae Curves, I', II' & B' represent this relation for their corresponding curves. From these it is seen that, B', is an accurate straight line while, I', deviates slightly and II' a greater amount.

From the character of the test curve, B', it may be inferred that the curve B is a correspondingly accurate parabola. It may also be observed that the straight line intercepts the axis at a point below the origin corresponding to the mechanical friction as in previous tests.

The speed current curves obtained previously were accurate curves through the range of speed employed in the practical application of the meter. These curves however show that the law of the meter holds through an extended range and further that the self-braking effect is the principal factor in the deviation from this law.

Transformer.   The addition of the transformer in the modified form introduces slight complications into the action of the meter. The two principal effects are the regulation and power factor of the device.

The regulation of the transformer is slight and as the voltage on the mains varies by only a few per cent, the load and consequently the primary and secondary drops are effected in a correspondingly small amount and as the total drop is itself slight the effect under normal conditions is very slight.

The use of the transformer in the modified form of the meter involved a certain secondary power factor which by a prop-
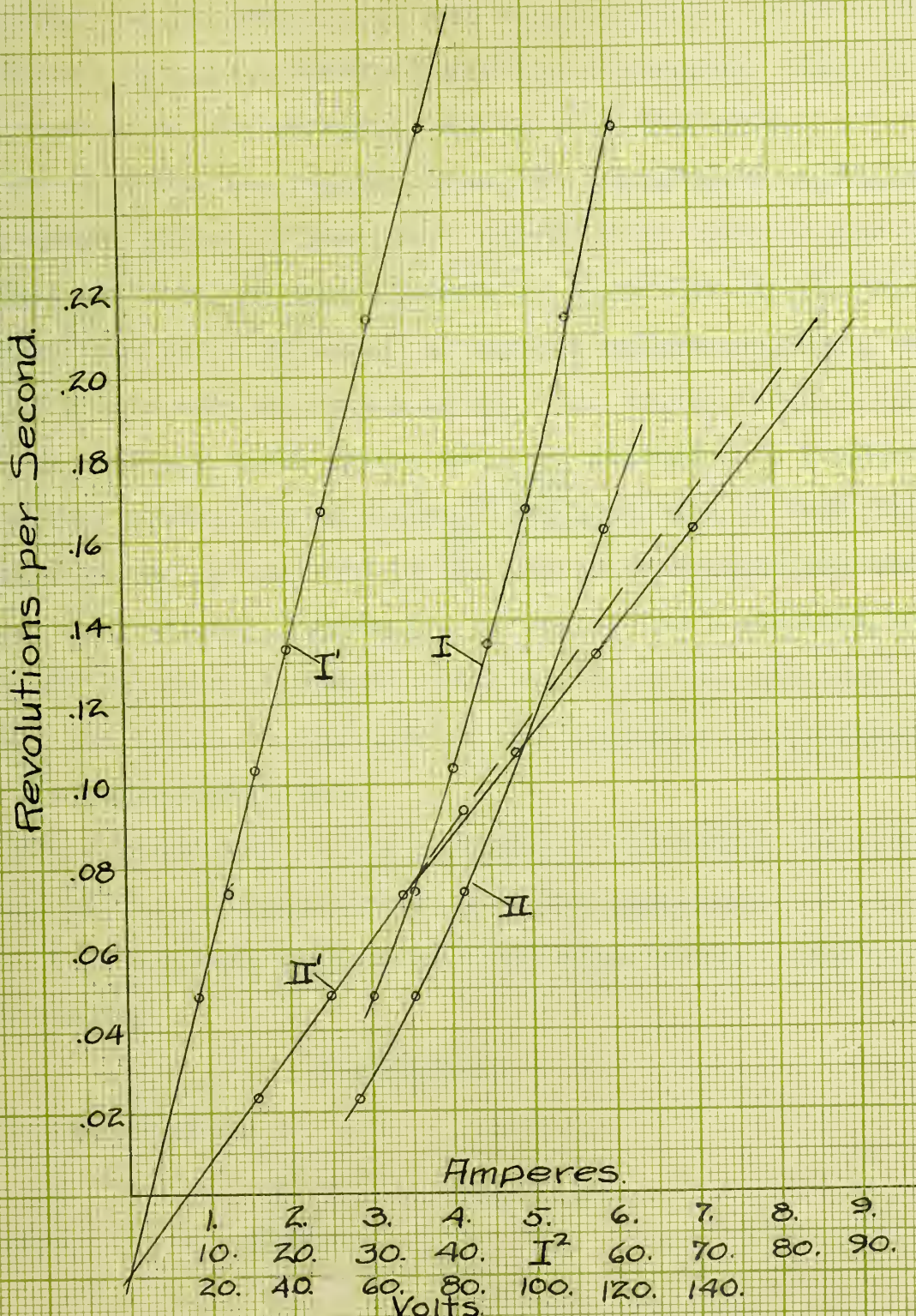
TRANSFORMER REGULATION. (Fig.32.).

TEST #8.

er arrangement of constants was made equal to that of the
circuit to be metered. The condition for this equality was
that the resistance of the motor winding was negligible com-
pared with its inductance and that the inductance of the trans-
former was negligible compared with its resistance. Each of
these conditions are obtained only in the ideal case.

To determine the regulation of the transformer, ordinary
speed torque curves of one motor element were obtained and then
a second set from the same element, in which the current was
supplied from the transformer at various primary voltages. The
results of the test are shown in Fig. 32. Curves I and I' repre-
sent the ordinary speed curves and are plotted with current
and current squared as abscissae. Curves II and II' are the
second test and are plotted to the impressed voltage and its
square, respectively. Curve,I', is seen to be a straight line,
while, II' is seen to drop off slightly from this straight line
showing the effect of the drop at the terminals of the secondary
from its assumed magnitude.

The "elimination of the transformer inductance" is accom-
plished by the reversed direction of the series current in one
coil. Following are data taken from tests on the transformer and
motor winding.

<center>Transformer.</center>

One secondary alone

X	r
.2684	.0443

Two secondaries opposed.   Each

X	r	X	r
.0215	.0896	.0108	.0448

Motor Winding.

X	r
.1297	.0573

Using the above values, the angles θ_1 and θ_2 as denoted previously were computed as follows. See page   and Figs 7 and 8

$A = E\,(g_1 + j\,b_1)$

$C = E\,(g_2 + j\,b_2)$

$I = E\left[(g_1 + g_2) + j(b_1 + b_2)\right.$

$$g_1 = \frac{r}{r_1^2 + X_1^2} = \frac{.0573}{.0573^2 + .1295^2} = 2.87$$

$$b_1 = \frac{X_1}{r_1^2 + X_1^2} = \frac{.1295}{.0573^2 + .1295^2} = 6.47$$

$$g_2 = \frac{r_2}{r_2^2 + X_2^2} = \frac{.0448}{.0448^2 + 0122^2} = 20.77$$

$$b_2 = \frac{X_2}{r_2^2 + X_2^2} = \frac{.0122}{.0448^2 + .0122^2} = 5.66$$

$$A = E\ (2.87 + j\ 6.47) = E\sqrt{2.87^2 + 6.47^2} = 7.03\ E$$

$$C = E\ (20.77 + j\ 5.66) = E\sqrt{20.77^2 + 5.66^2} = 21.55\ E$$

$$I = E\ (23.64 + j\ 12.13) = E\sqrt{23.64^2 + 12.13^2} = 26.57\ E$$

$$\cos\theta_1 = \frac{A^2 + I^2 - C^2}{2\ A\ I} = .777$$

$$\theta_1 = 51°$$

$$\tan\theta_2 = \frac{X_1 + X_2}{R_1 + V_2} = \frac{.1295\ .0122}{.0573\ .0448} = \frac{.1417}{.1021} = 1.387$$

$$\theta_2 = 54°\ 15'$$

$$\theta_2 - \theta_1 = 3°15' = 3.25° = \text{angular error.}$$

Note:— $\cos 3.25° = .9984$

It is seen from these figures that although it is not practical to construct a machine which neither has inductance in the transformer nor resistance in the motor winding, it is possible to accomplish the same effect to a close degree of precision by proper adjustment of these constants.
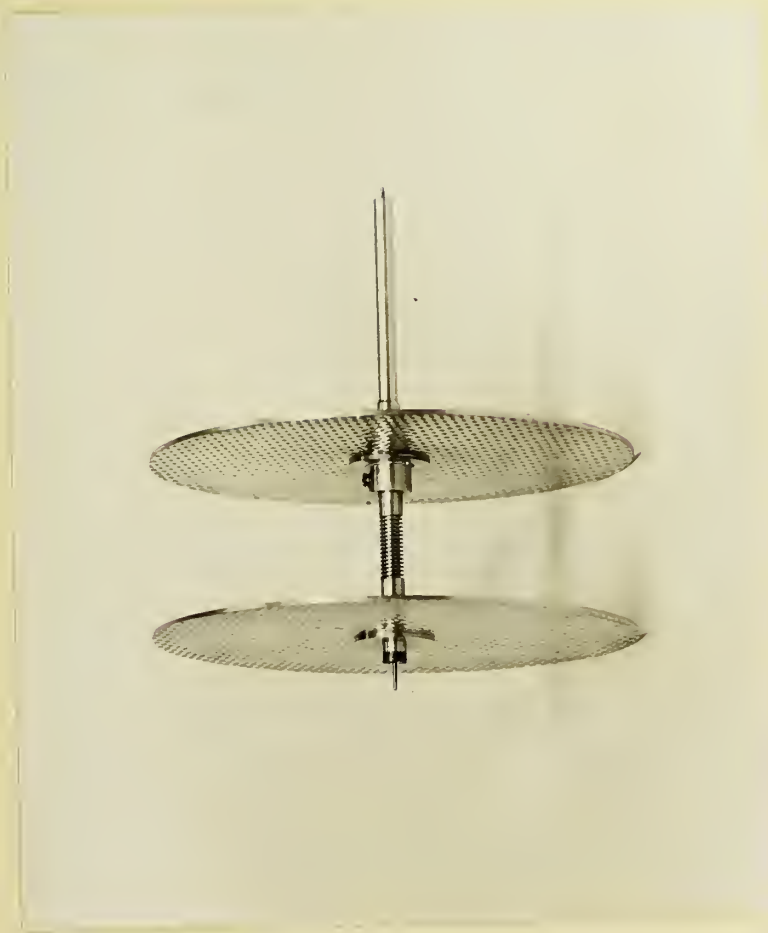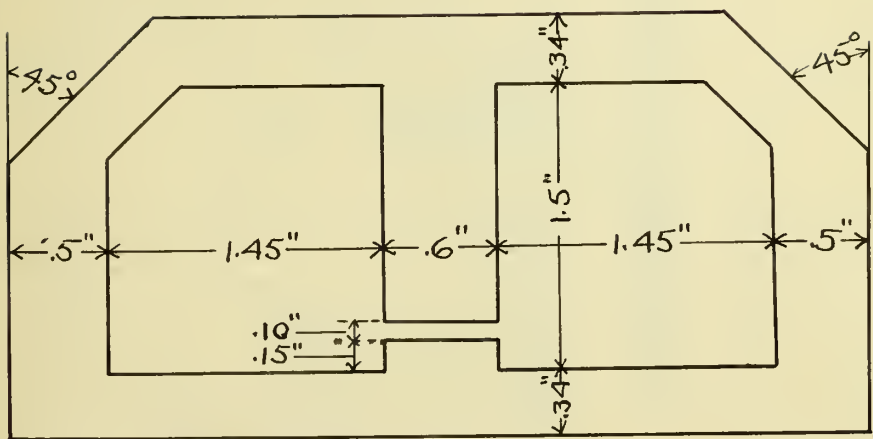
40 thicknesses —.015" Transformer Iron.

CORE STAMPINGS. (Fig. 33.)

.03" Transformer Iron.
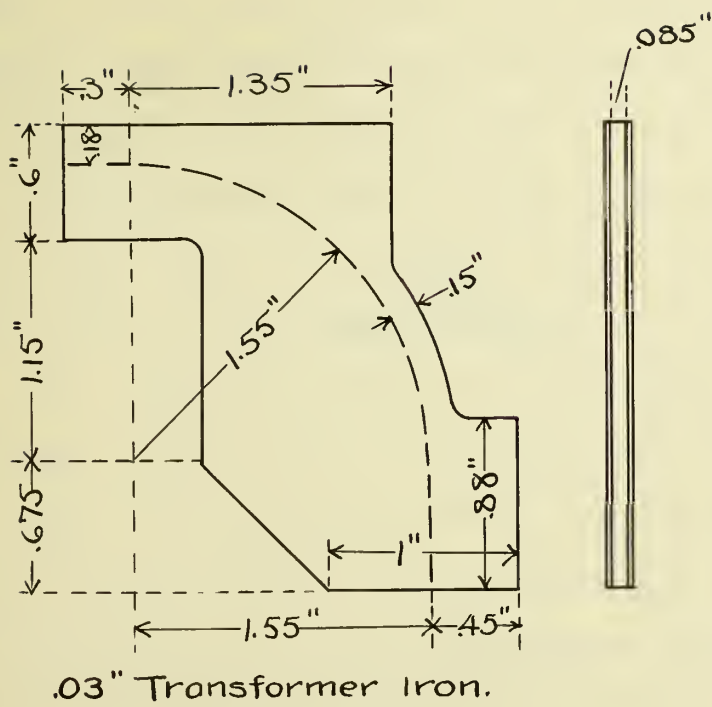
SKETCH OF WING. (Fig. 34.)

Description of Model #25.

In the development of this new meter a number of experi-
mental models were constructed. These were of the same general
construction and varied principally in the matter of details.
The first models were however of the two winding type while the
latter ones used the transformer to combine the e.m.f. and cur-
rent factors. It is one of these latter that was used in the
previous work. The machine being designated as Model #25.

The cores of the two elements which are identical are com-
posed of .015" transformer sheet; built up to a thickness of
.6 in. The center limb of the shell (Fig.33) which carries
the winding is also 6 in. wide, making an air gap .36 sq.in.
by .1 in. long. These two cores are mounted on a steel plate
upon which the various parts of the mechanism are mounted.

The rotating system consists of two aluminum discs 3 in. in
diameter and .035 in. thick, mounted about 1.1 in apart, upon
an aluminum staff about .1 in. in diameter. This staff is
supported by a "rotated jewel" the pivot being held in a spring
seat in the end of the shaft. The upper bearing is merely a
cylindrical pivot turning in a hollow adjustment screw. The discs
are checkered to give them additional stiffness.

A bracket on the front plate serves to support each of the
two wings. (Fig.34) These wings are made up of .03 in. trans-
former iron and have a space air gap of .085 in. on which the
disc revolves. Each wing is held on its bracket so that it may
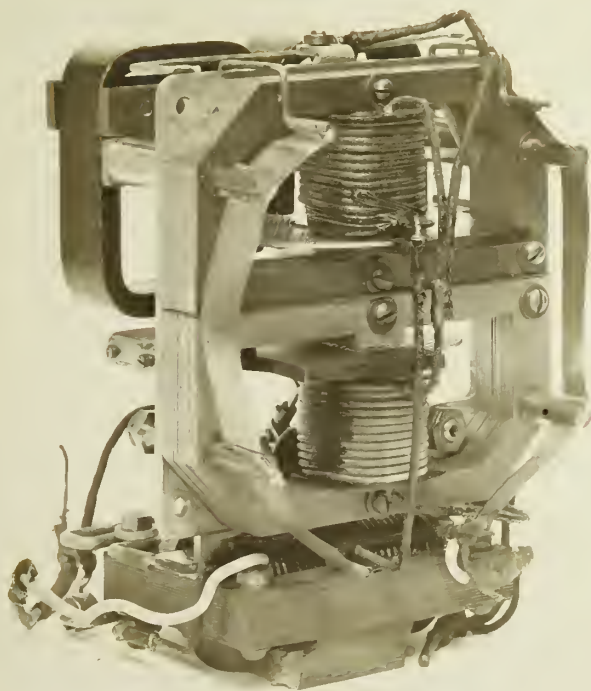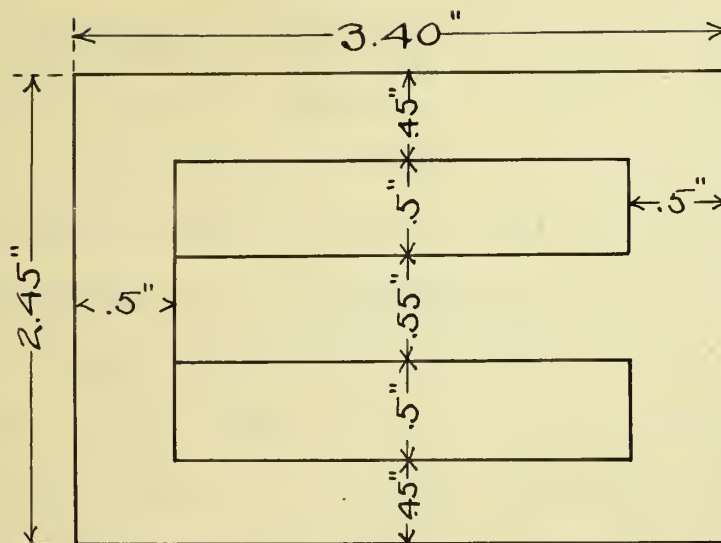
36 thicknesses — .015" Transformer Iron.

TRANSFORMER STAMPINGS. (Fig. 35)

be drawn in or out to provide the proper wing area.

At the bottom of the front plate is mounted the trans-
former (Fig.35) which is of the shell type. The core is built
up of .015 transformer iron. The magnetic circuit having but
a single joint. The primary which takes the voltage of the
mains is wound on the inside. While the low pressure secondaries,
of which there are two, are on the outside.

The circuits which are shown in Fig.34 contain three ad-
justable resistances, which are provided to compensate for any
difference in the two elements. It will be noticed also in the
diagram that the windings of the transformer are arranged so
that the shunt current passes through the two halves in oppo-
site directions; the inductive effect on the transformer and
the shunt circuit itself being practically nil The resistances
at,m and n, are of German Silver and are introduced to proper-
ly adjust the valve of the shunt snd series currents in the mo-
tor winding and also to adjust the auxiliary power factor.

The meter is provided with the usual retarding magnets
and recording train. The magnets are two in number and are
placed with poles reversed, thus localizing the eddy currents.
These magnets are placed on the upper disc of the differential
system. The magnets are adjusted by magnetic shunt which is
arranged to give a wide variation in the flux of the magnets.
The recording train is connected to the shaft by a worm gear and
consists of a series of dials. To the last of these is geared
the rotated jewel which makes one revolution to about one
million of the disc.
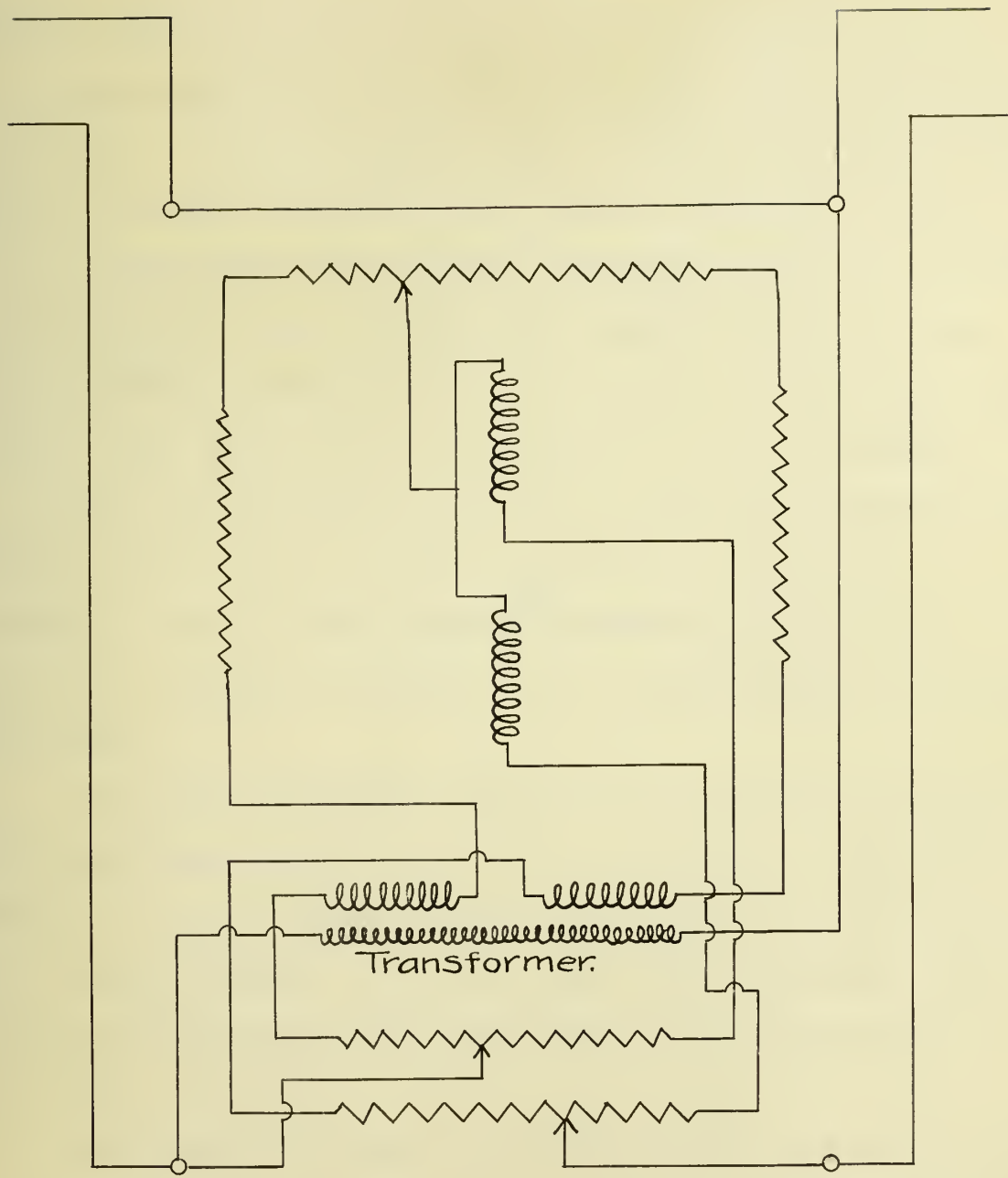
ARRANGEMENT OF MODEL #25. (Fig. 36.)
(Back View.)

The whole device is very compact and is contained in a metal case about $4^1/2$ in. x 5 in. x $6^1/2$ in.

Adjustments.

The adjustments of the meter are three (not including the adjustment of the constant). These three adjustments are provided to balance inequalities of the two elements. They are (1) To adjust wing areas so that the same current will produce equal torque in each element. (2)To adjust circuit so that shunt current is equal in the two windings. (3) To adjust circuit so that series current is equal in the two windings.

The first adjustment is made by removing the upper sliding contact when the two elements are directly in series. If a current is sent through the meter, the wings may be set in or out until equilibrium results and the discs rotate in neither direction.

The second adjustment consists in applying line voltage to the transformer primary, when with no current in the series terminals the upper sliding contact may be placed so that equilibrium again results. It is seen that this adjustment cuts resistance out of one of the transformer circuits and puts it in the other.

The third adjustment consists in sending a current through the meter when no pressure is applied to the transformer terminals; the two lower sliding contacts being so placed that equilibrium is again reached. This adjustment varies the resistance in parallel with the motor windings; equilibrium being reached

Performance Tests. (Fig. 37.)

#1 — 61∿ — P.F = 1.0
#2 — 61∿ — P.F = 1.0

Revolutions per Second per Watt.

Hekto-Watts.

Test #6.

a.— 61 ∿ - P.F =1.0.
b.— 69.3∿ - P.F=1.0.
C.— 47.8∿ - P.F=1.0
d.— 61.0∿ - P.F=.5

Revolutions per Second per Watt.

.00070
65
60
55
50
45

1.   2.   3.   4.   5.   6.   7.   8.   9.   10.

Hekto-Watts.
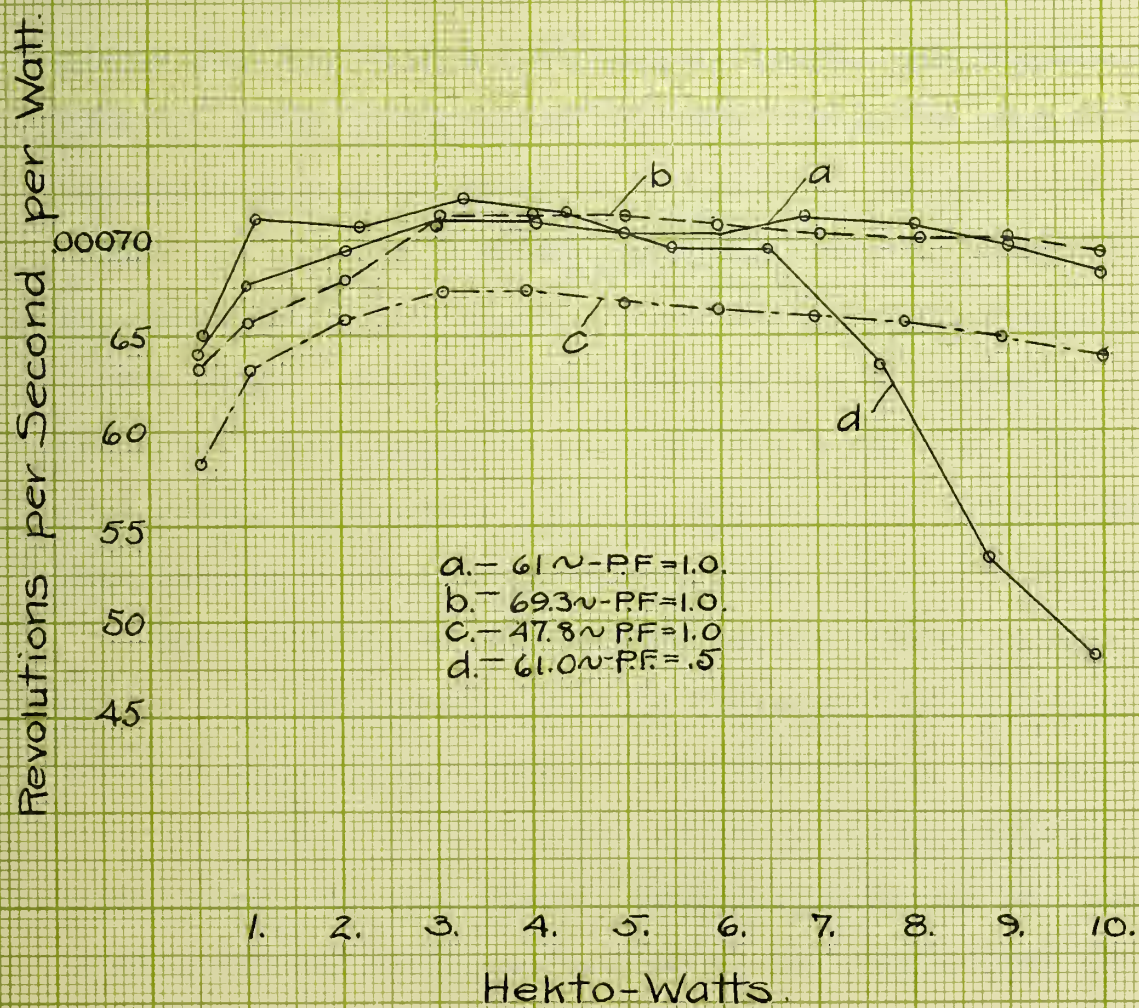
Performance Tests. (Fig. 38)

Test #27.

when the two elements are equally shunted.

Commercial Tests.

The performance of a meter under ordinary load conditions is the most important indication of its value as a commercial instrument. Accuracy being the essential feature in such a device.

A few runs were made on Model #25 to determine its accuracy under load conditions both at normal power factor and frequency and then with a variation of these. The results of such runs are shown in Fig. 37 & 38. The curves are seen to have a characteristic shape which is effected but little by frequency though the curve is displaced somewhat for the lower frequency. The curve at .5 P.F. shows a striking decrease of rate for the larger loads. The meter was found to run on a load of 6 watts. The above tests were made with the train removed. The meter runs either backward or forward, the constant varying but little when the meter is reversed by reversing e.m.f. or current.

The deviation of the constant under varying loads is a matter of adjusting the magnetic constants and would not appear to be an irremediable fault of the instrument. On the other hand the extreme sensitiveness of the meter in registering light loads; the fact that it does not creep and further runs equally well in either direction are advantages inherent in the device.
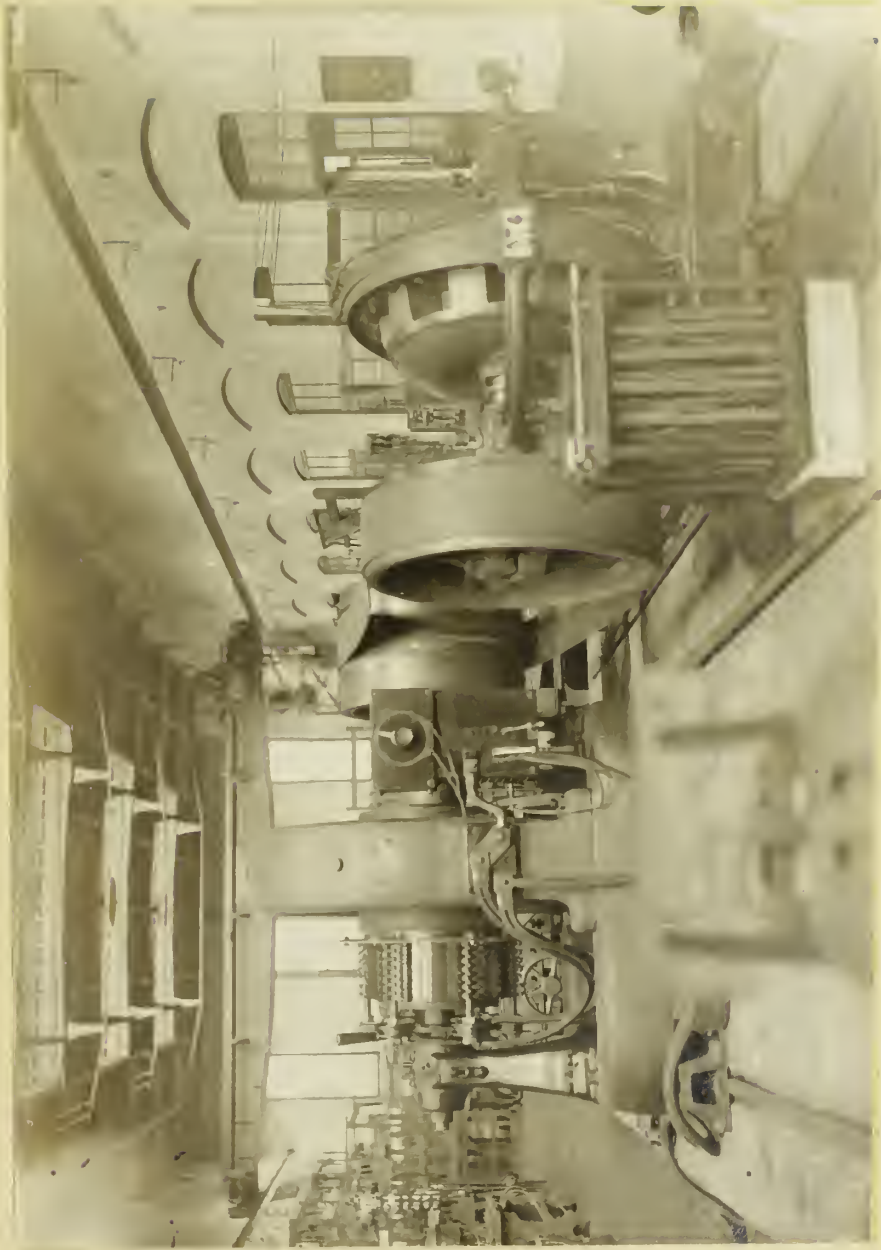
DATA

Test #8

Speed Voltage Curves 60 ᴨᴠ

(Fig.32)

E Volts	V Rev.	S Sec.	r/s	E^2	$r/s/E^2$
118.3	4	24.7	.1620	14050	
107.9	4	30.5	.1312	11650	
98.0	3	27.8	.1079	9620	
82.6	2	27.3	.0733	6820	
70.6	2	41.5	.0482	5000	
56.4	1	43.6	.0230	3180	

Test #8

Speed-Current Curves   60 ∿

(Fig.32)

I Amp.	v Rev.	S Sec.	r/s	I^2	$r/s/I^2$
2.99	2	41.4	.0484	9.00	.00538
3.52	2	27.1	.0738	12.40	.00596
4.02	3	28.9	.1038	16.20	.00641
4.49	4	29.9	.1339	20.20	.00663
4.98	5	29.5	.1672	24.80	.00675
5.50	7	32.7	.2139	30.30	.00706
6.10	8	30.8	.2596	37.20	.00698
6.50	9	29.3	.3072	42.35	.00726
7.06	10	27.6	.3620	50.00	.00724
7.61	12	28.5	.4210	58.00	.00727
8.20	16	33.0	.4850	67.20	.00722
9.10	16	27.4	.5840	82.90	.00704

Test #11 a

Torque-Current 30 ⋏⋎

(Fig.20)

I Amp.	d Mn	P G.	T G.cm	I^2	K $\dfrac{P}{I^2}$
2.32	.4	.0247	.100	5.40	.00457
3.12	1.0	.0619	.248	9.75	.00619
3.74	1.5	.0927	.372	14.00	.00662
4.10	1.9	.117	.468	16.85	.00695
4.32	2.3	.142	.563	18.70	.00760
5.00	3.0	.185	.740	25.00	.00740
5.90	4.3	.266	1.064	34.85	.00764
6.54	5.0	.309	1.236	42.80	.00725
8.30	8.5	.525	2.100	69.00	.00761
8.50	9.1	.556	2.224	72.40	.00768

$$P\ (g)\ =\ \frac{d\ (mm)}{16.17}$$

$$T\ (g.cm) = 3.99\ P(g)$$

Test #11b

Speed–Current   30 ⋈.

(Fig.24)

I Amp.	r Rev.	s Sec.	$\frac{r}{s}$	I	$\dfrac{K}{\dfrac{r}{s}}{I}$
2.07	1	60.3	.0166	4.29	.00386
2.72	1	25.1	.0398	7.42	.00537
3.46	2	28.5	.0702	12.00	.00585
4.22	3	27.0	.111	17.86	.00622
4.98	4	24.5	.163	24.80	.00658
6.43	7	25.6	.273	41.40	.00659
8.45	10	23.1	.433	71.50	.00606
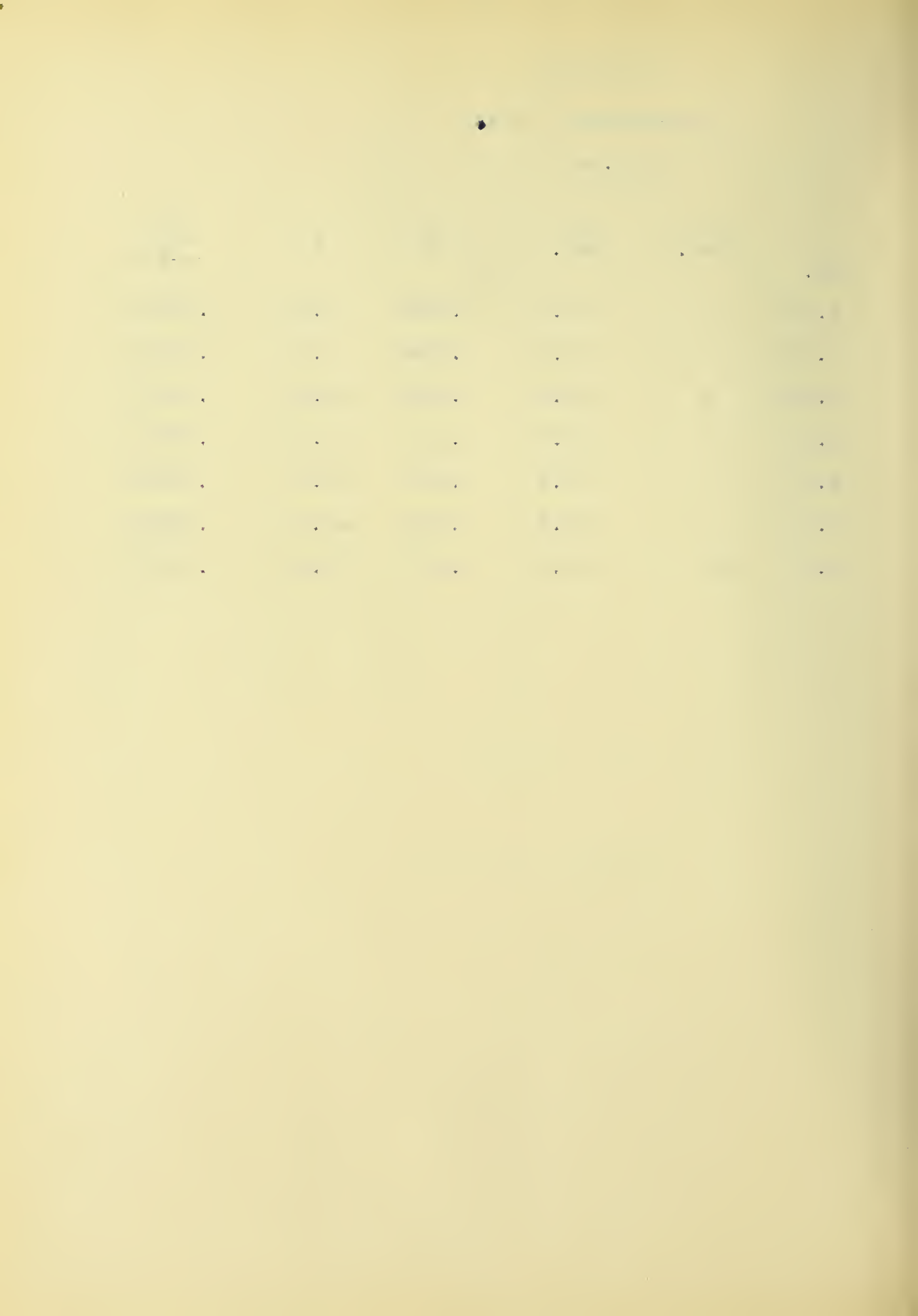
Test #11 c.

Torque—Current 45 v.

(Fig.21)

I Amp.	d Mm	P G.	T G.cm	I^2	$\dfrac{K_P}{I^2}$
2.00	1.1	.068	.272	4.00	.0170
2.72	2.0	.124	.496	7.42	.0167
3.19	2.1	.130	.520	10.2	.0127
3.69	2.9	.179	.716	13.6	.0131
4.02	3.3	.204	.811	16.20	.0126
4.43	4.5	.278	1.112	20.10	.0138
5.30	6.4	.396	1.534	28.15	.0140
6.55	10.2	.631	2.524	42.90	.0147
8.50	17.1	1.057	4.224	72.30	.0146

$$P\ (g) = \frac{d(mm)}{16.17}$$

$$T\ (g.cm) = 3.99\ P(g)$$

Test #11 d.

(Fig.25)

Speed—Current. 45 ʌl

I Amp.	r Rev.	s Sec.	$\frac{r}{s}$	I	K $\frac{\frac{r}{s}}{I}$
1.92	2	39.7	.0504	3.70	.0137
2.52	3	34.8	.0863	6.36	.0157
2.96	3.5	30.0	.1167	8.73	.0133
3.23	4	28.2	.1418	10.44	.0136
4.06	6	30.7	.1952	16.53	.0118
4.92	10	29.4	.3404	24.30	.0140
5.83	16	34.2	.4680	34.00	.0138
6.98	24	37.3	.6440	48.85	.0132
9.05	30	31.0	.9680	81.20	.0119

Test #12 d

Speed-Current   71.4 v

(Fig.27)

I Amp.	r Rev.	S Sec	$\frac{r}{s}$	I^2	$\frac{\frac{r}{s}}{I^2}$
1.20	1	32.8	.0352	1.44	.0244
1.42	2	40.6	.0493	2.02	.0246
1.80	2	26.6	.0753	3.24	.0232
2.32	4	30.1	.1329	5.38	.0247
2.71	6	32.1	.1368	7.34	.0255
3.22	8	30.0	.2662	10.36	.0257
3.67	10	28.6	.350	13.47	.0260
4.41	14	27.7	.506	19.45	.0260
5.03	18.66	28.1	.664	25.30	.0262
6.02	28	30.5	.918	36.24	.0253
6.96	30	24.6	1.219	48.44	.0252
7.90	40	26.6	1.562	62.41	.0251

Test No.14.a.

(Fig.22)

Torque—Current   60 λ

I Amperes	d Mm	P g.	T G.Cm	I	K $\dfrac{G.Cm}{I}$
.97	.25	.015	.062	.94	.0660
1.50	.53	.033	.135	2.25	.0600
1.97	1.43	.088	.354	3.88	.0913
2.51	1.80	.111	.444	6.30	.0705
2.99	2.70	.167	.668	8.94	.0748
3.72	4.40	.272	1.088	13.84	.0793
4.44	6.30	.389	1.556	19.71	.0790
5.05	8.80	.544	2.176	25.50	.0854
6.04	12.90	.798	3.192	36.48	.0875
7.02	17.60	1.089	4.356	49.28	.0885
7.85	22.30	1.379	5.516	61.62	.0897
8.80	27.50	1.700	6.800	77.44	.0880
9.90	35.00	2.162	8.648	98.01	.0883

d' = deflection for 1g. = 16.17 mm.

$$T = \frac{b}{a} \times v \times P = 3.99\ P$$

$$P = \frac{d}{16.17}$$

$$T = \frac{b}{a} \times v \times \frac{d}{16.17} = .247\ d\ (G.Cm)$$

Test No.14.b.

(Fig.26)

Speed—Current 60 ɴ.

I Amp.	r Rev.	s Sec.	$\frac{r}{s}$	I	$\frac{\frac{r}{s}}{I}$
1.36	1	45.2	.0221	1.85	.01194
1.79	2	39.5	.0507	3.20	.01584
2.08	3	39.8	.0754	4.33	.01739
2.37	4	36.1	.1108	5.62	.01970
3.14	6	29.5	.2032	9.86	.02062
3.78	8	26.3	.3040	14.29	.02126
4.61	12	26.5	.4530	21.25	.02130
5.21	16	27.6	.580	27.14	.02138
6.19	20	25.1	.7 7	38.31	.02062
7.10	24	23.6	1.017	50.41	.02017
7.90	30	25.3	1.146	62.41	.01836
9.12	36	22.8	1.490	83.17	.01793

Torque Frequency Curves.

(Fig.28)

Torque.

	2 amp.	4 amp.	6 amp	8 amp.
30 $\sim$	.05	.44	1.04	1.93
45 $\sim$	.19	.85	2.07	3.75
60 $\sim$	.27	1.25	3.10	5.67

Data taken from Figs 20,21, & 22.

Test #20

Speed Current   60 $\curvearrowright$

Iron

I Amp.	V Rev.	S Sec.	V/S	I^2	$V/S/I^2$
1.30	1	31.0	.0322	1.69	.0191
2.00	3	44.0	.0683	4.00	.0171
2.85	5	35.4	.1752	8.12	.0216
3.56	7	32.2	.2173	12.67	.0172
4.37	10	30.6	.327	19.10	.0172
5.39	18	36.6	.482	29.05	.0166
6.70	28	38.2	.582	44.89	.0130

Brass.

I Amp.	V Rev.	S Sec.	V/S	I^2	$V/S/I^2$
1.35	1	31.6	.0316	1.82	.0174
1.97	2	31.4	.0638	3.88	.0164
2.70	4	33.4	.1198	7.29	.0164
3.70	8	36.2	.2210	13.69	.0162
4.73	12	33.2	.361	22.37	.0162
5.91	20	36.4	.550	34.93	.0157
6.98	24	32.4	.742	48.72	.0152

Test #23

"Test" Curves

(Fig.29)

2 amp. Freq. ν	ν^2	Speed $\frac{r}{s}$	K $\frac{\frac{r}{s} \times 10^{-6}}{v_2}$	4 amp. Freq. ν	ν^2	Speed $\frac{r}{s}$	K $\frac{\frac{r}{s} \times 10^{-6}}{\nu^2}$
69.5	4830.	.1132	23.4	69.5	4830.	.492	.102.
37.0	1369.	.0385	28.1	37.0	1369.	.1876	137.
27.2	739.	.0160	21.7	26.8	718.	.1037	144.
				15.3	234.	.0243	104.

6 amp. Freq. v	v^2	Speed r/s	K $r/s/v_2 \times 10^{-6}$	8 amp. Freq. v	v^2	Speed $\frac{r}{s}$	K $\frac{\frac{r}{s} \times 10^{-6}}{v^2}$
69.5	4830.	1.096	230.	69.5	4830.	1.792	371.
37.0	1369.	.426	311.	37.0	1369.	.692	506.
26.7	713.	.234	328.	25.7	660.	.366	554.
18.0	324.	.1092	338.	21.5	462.	.250	541.
9.3	86.5	.0274	317.	16.8	282.	.1488	527.
				12.2	149.	.0667	447.
				9.2	84.6	.0309	366.

Speed Frequency Curves

(Fig.29)

Speeds.

	2 amp.	4 amp.	6 amp.	8 amp.
30 $\sim$	.014	.100	.239	.397
45 $\sim$	.057	.218	.495	.804
60 $\sim$	.067	.340	.750	1.22
71.4 $\sim$	.098	.419	.917	1.52

Data taken from Fig's 24,25,26, & 27.

Test # 23

Speed Frequency

(Fig.30)

2 amp.		4 amp.		6 amp.		8 amp.	
Freq.	Speed	Freq.	Speed	Freq.	Speed	Freq.	Speed
$\sim$/s	r/s	$\sim$/s	r/s	$\sim$/s	r/s	$\sim$/s	r/s
69.5	.1132	69.5	.4920	69.5	1.096	69.5	1.792
37.0	.0385	37.0	.1876	37.0	.4260	37.0	.6920
27.2	.0160	26.8	.1037	26.7	.2340	25.7	.3662
		15.3	.0243	18.0	.1092	21.5	.2500
				9.3	.0274	16.8	.1488
						12.2	.0667
						9.2	.0309

Test #26 a.

Performance Tests   61 𝒩

E	P.F.	W	R	S	R/s/W
110	1.0	102	4	63.8	.000615
		203	10	76.2	647
		299	12	60.4	720
		400	16	58.6	683
		504	20	58.4	682
		599	24	58.8	687
		706	30	62.0	689
		704	30	62.4	687
		802	36	65.4	686
		892	36	59.4	681
		998	40	59.6	673

Test #26 b

Performance Tests   61 ∿

E	P.F.	W	R	S	R/S/W
110	1.0	53	2	60.4	.000625
		102	4	60.6	647
		200	10	75.6	662
		301	12	59.8	668
		402	18	64.6	693
		507	20	57.8	684
		600	24	58.6	683
		697	28	58.6	685
		810	36	65.4	680
		910	40	65.2	675
		1020	40	64.6	609

Meter runs on 10 watts.
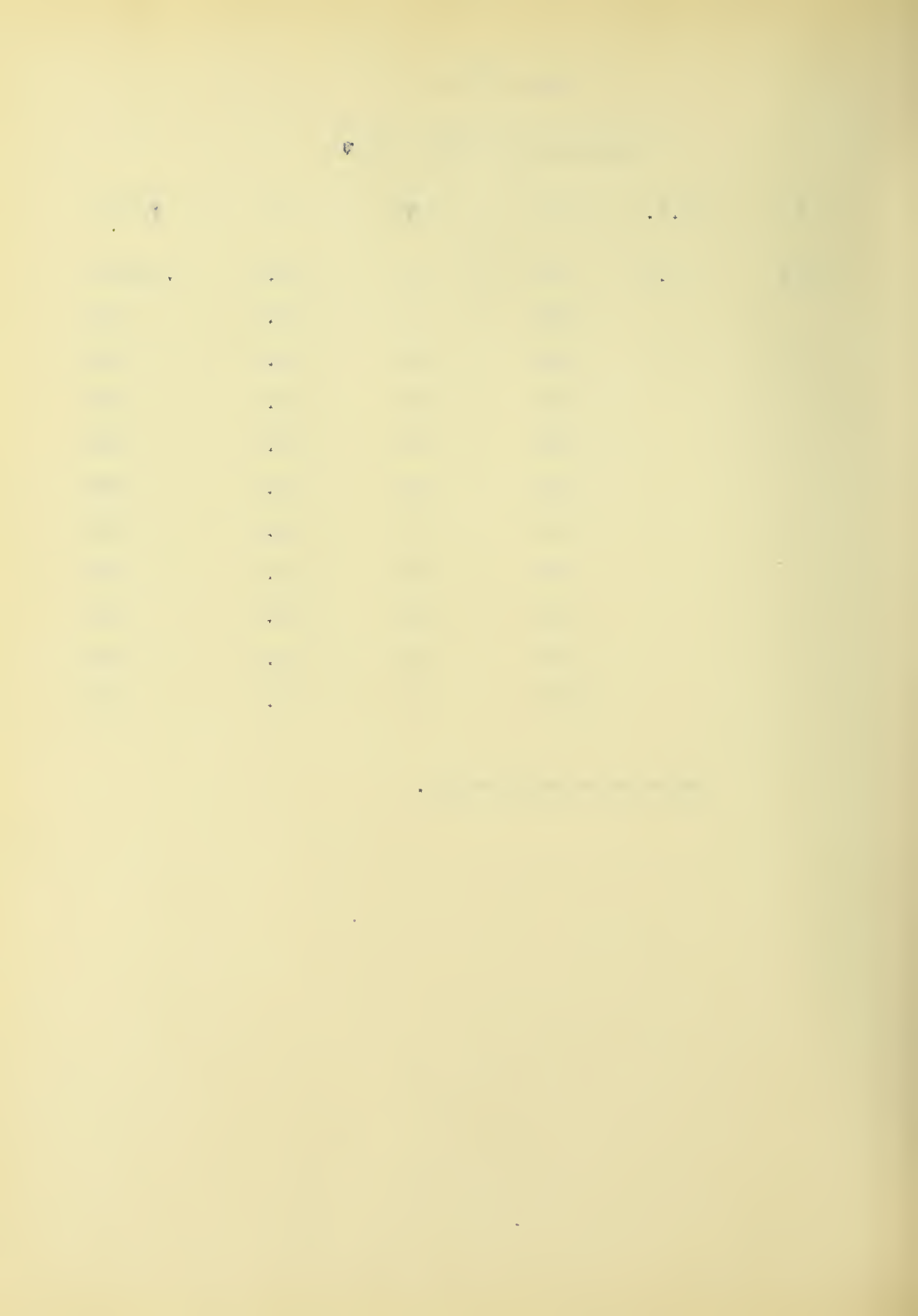
Test # 27 a

Performance Tests   61 𝓂𝓁

E	P.F.	W	R	S	R/s/W
110	1.0	52	2	60.2	.000640
		102	4	58.0	676
		203	8	56.8	694
		302	12	55.8	713
		406	16	55.8	708
		400	16	56.2	712
		L-400	16	57.8	693
		500	20	57.0	702
		600	24	56.8	703
		690	28	57.2	711
		803	32	56.4	707
		900	36	57.4	696
		1007	40	58.4	680
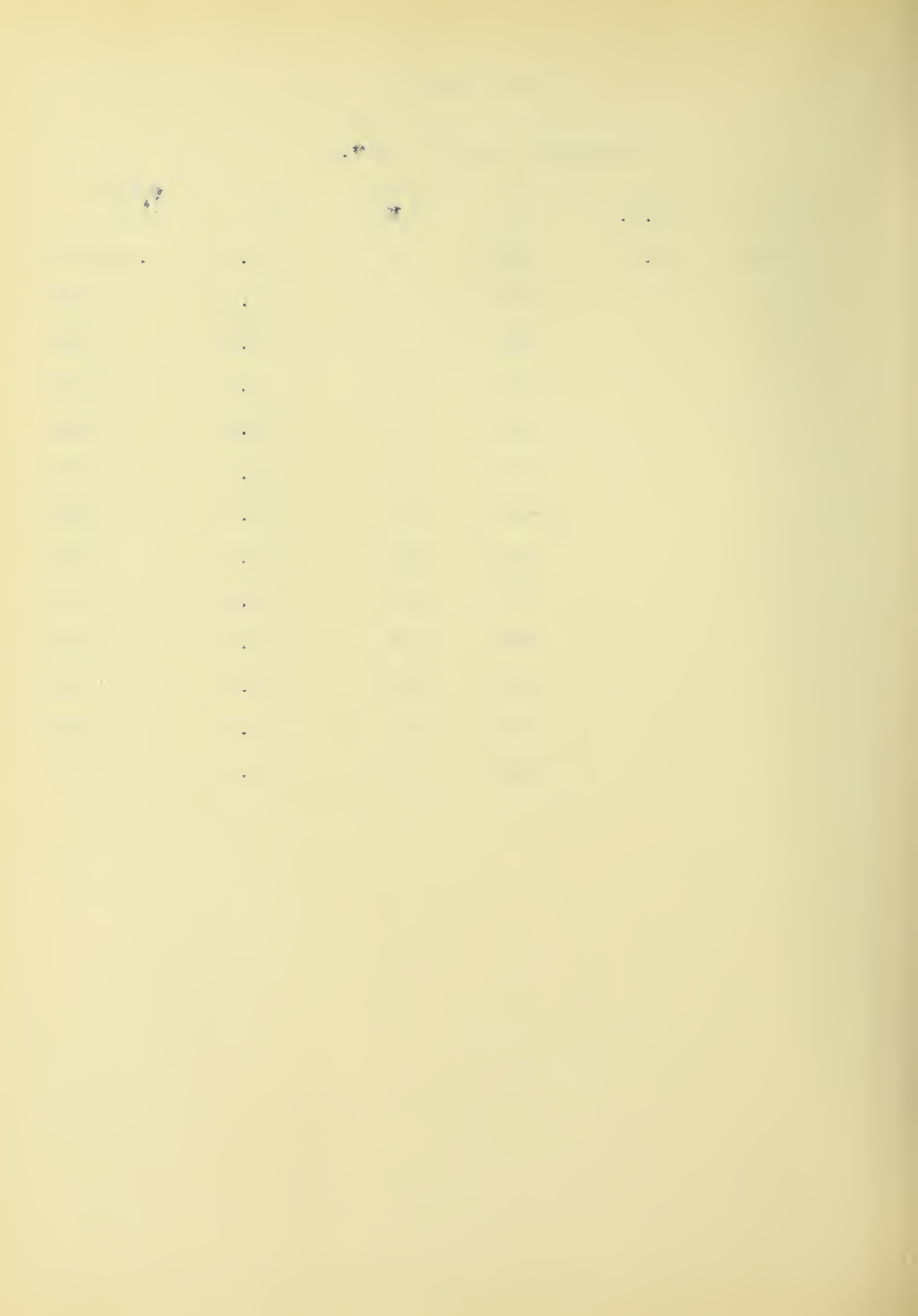
Test #27 b

Performance Tests 69.3 $\curvearrowright$

E	P.F.	W	K	S	K/s/w
110	1.0	52	2	60.8	.000633
		102	4	59.8	656
		203	8	58.0	679
		300	12	56.2	713
		400	16	56.2	713
		500	20	56.6	712
		592	24	57.2	708
		702	28	56.8	703
		807	32	56.6	702
		900	36	57.2	701
		997	40	58.0	692
		1240	25	29.3	588

Meter runs on 10 watts.
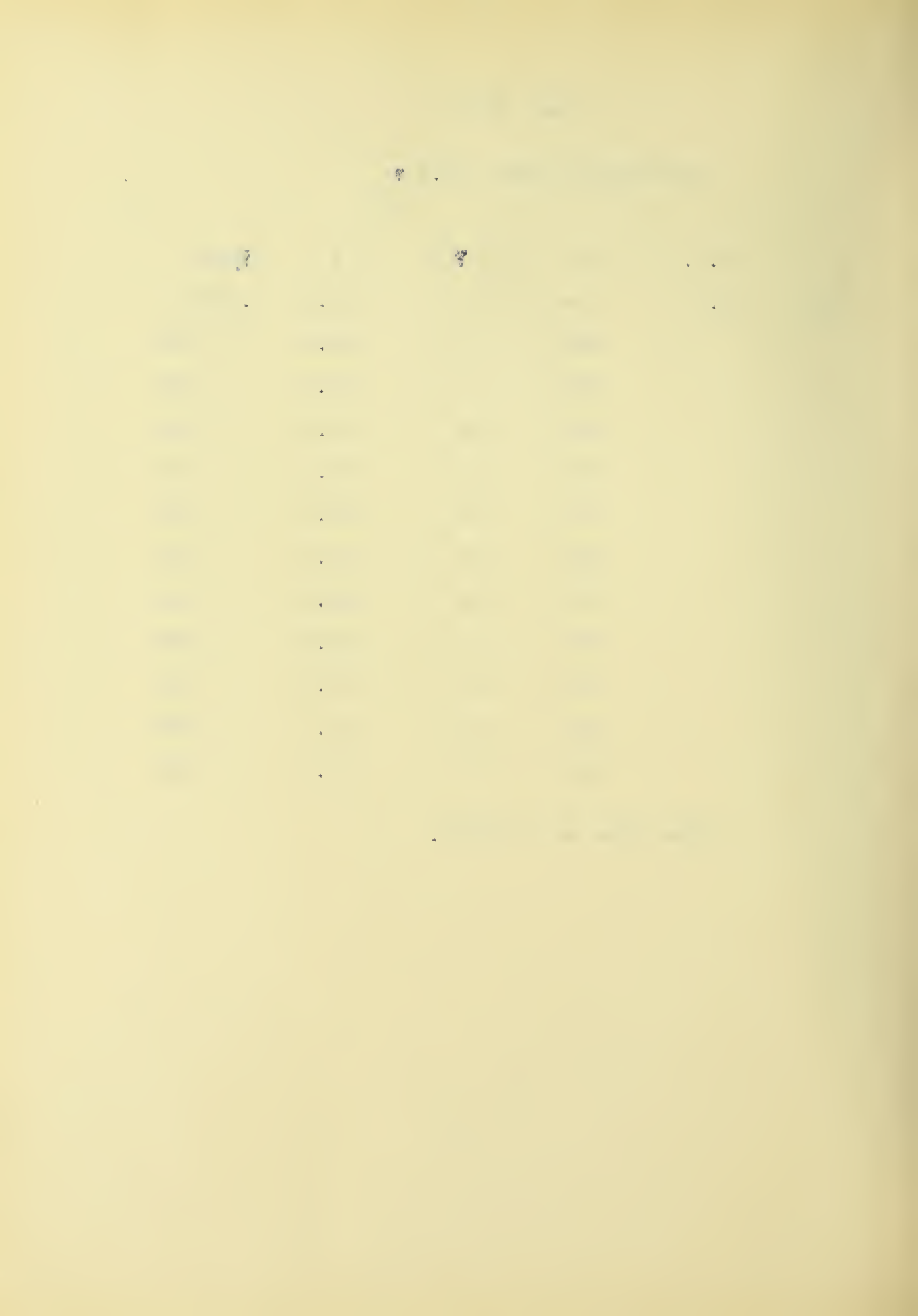
Test #27 c.

Performance Tests   47.8 λ.

E	P.F.	W	R	S	R/S/W
110	1.0	52	2	66.0	.000583
		102	4	62.0	633
		203	8	59.8	658
		305	12	58.6	673
		396	16	60.0	674
		498	20	60.2	668
		597	24	60.6	663
		698	28	60.8	660
		792	32	61.6	656
		895	36	62.0	649
		1000	40	62.6	640
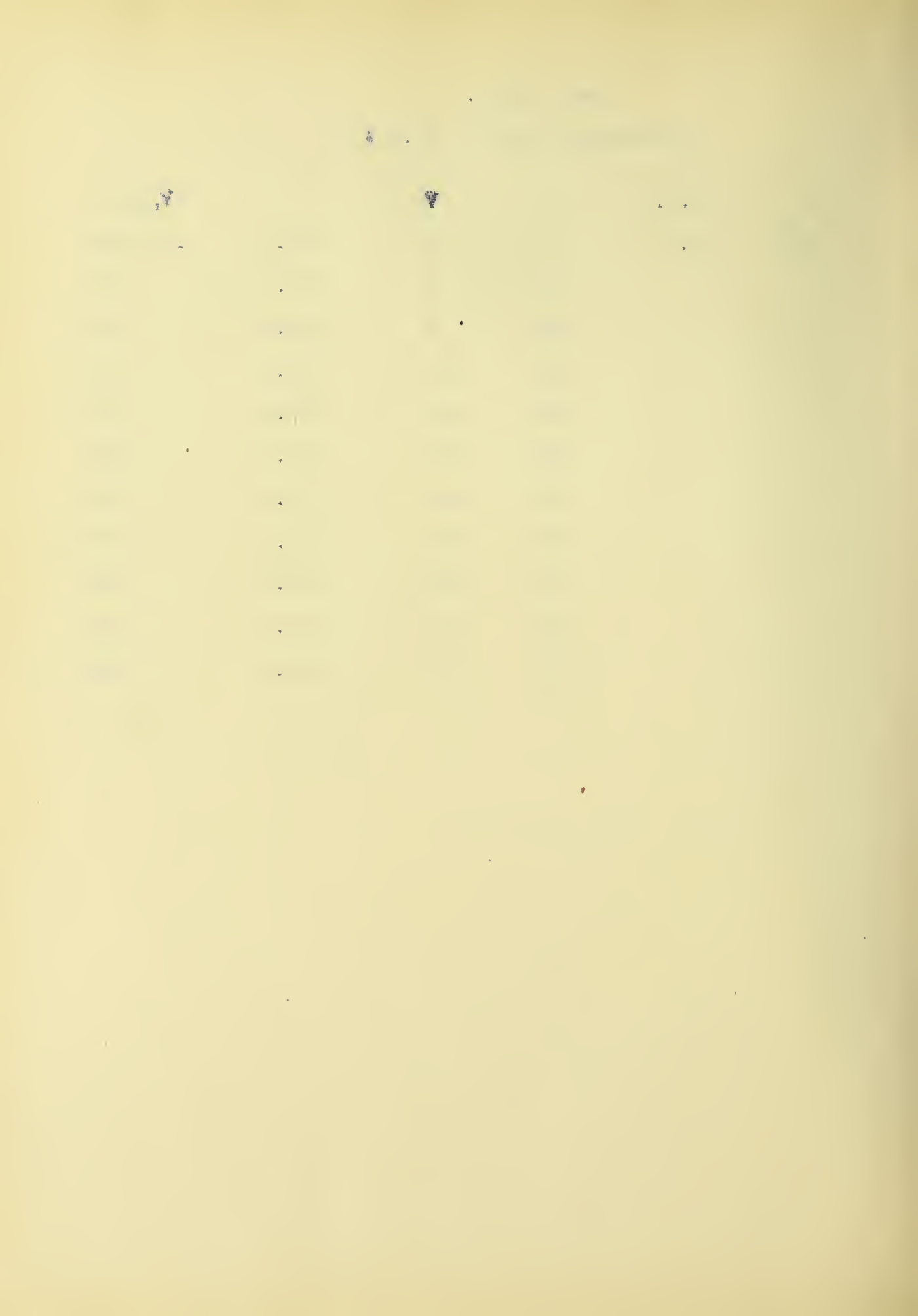
Test #27 d.

Performance Tests   61.0 N.

E	P.F.	W	K	S	K/s/w
110	.5	57	2	54.0	.000650
		112	4	50.0	713
		223	8	50.8	707
		330	12	50.4	722
		436	18	57.2	715
		550	22	57.6	695
		650	26	57.4	697
		770	30	61.5	634
		882	26	55.4	533
		990	36	75.6	482